ICT Support in Mathematics and Physics Integrated Teaching Based on Modeling Process

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Abstract

This article presents a part of the scientific missions of the Inter-Tetra project: Teaching based on modeling process has been published from many didactic researchers for mathematics, physics and math-physics integrated teaching. However, in many cases, the process of building model and the operating of the model is abstract to the students and even teachers. For a deeper understanding of modeling, the first part of the study will present an overview of modeling in mathematical and physical learning. The next part of this study will proposes the application at the steps of building model and operating model in teaching based on modeling process. In addition, this study also presents some examples of using these ICT tools in teaching some math-physics integrated topics and analyzes the advantages and disadvantages of using them.

1. Overview of modeling

1.1. Model and modeling

A Model is a thinking structure that reflects reality or process in the real world. Summaries of studies on psychology have shown, that the nature of the formation of knowledge is the construction of models that reflect the outside world (Chiu, 2000).

There are two types of models: mental models and conceptual models. The mental model is a structure of instant images that instantly reflect the outside world. Its structures will be analogical to real-world structures. The mental model helps us to quickly visualize, explain and predict the evolution of the real process that the model reflects. Therefore, understanding physical phenomena necessarily needs to build mental models.

However, the mental model is incomplete, unstable and often is not described clearly without limits and in general the differences between individuals (example: model depicting light: having a figure it is a sunny yellow environment, others imagine a beam of light rays, some envisioned as a continuous emission of light particles etc.). Mental models will be refreshed in new situations. The conceptual model is accurate, a complete representation of structures, scientifically validated and highly stable (Norman, 2014) For example, the light model shows that light has both the nature of electromagnetic waves and quantum properties. Thus, while the mental model is personal, internal, inadequate and highly flexible, the conceptual model is external, more complete and scientifically validated (Greca, 2000).

Learning modeling can be understood as the process of editing and refreshing internal mental models to best suit conceptual models (Nersessian, 1992). Thus, if understood in this sense, the process of learning scientific knowledge is playing "game of modeling" (Halloun, 1996).

1.2. Role of modeling in teaching

Three fundamental purposes of science teaching include: learning of science, learning about science and learning to do science (Hodson, 1992). To achieve this goals, students need to perform modeling corresponding to three types of modeling activities: Learning to understand models (model learning), learning to modify models to suit new purposes (model revision) and learn to create models (model production) (Justi, 2002).

However, according to Piaget's Constructivism, learning how to build knowledge (a model) is better than re-understanding a way of defining knowledge (a model) developed by others (Piaget, 1952). Therefore, the goal to help students build knowledge (models) for themselves should be of top concern.

1.3. Modeling cycle

Building models requires students to have analogical inferences and metaphorical reasoning (Coll, 2005). However, modeling is not just about showing how to use thinking techniques to create a model. Modeling should include all actions: From discovering the reasons for modeling, proposing the model's objectives, arguing for model recommendations, correctness and feasibility testing (Gilbert, 2012). If the testing process finds that the proposed model is not correct, the proposed process of modeling will have to be repeated, so modeling is cyclical. It can be perceived that these characteristics of modeling have many parallels with the way of building knowledge.

A modeling cycle that met these requirements was proposed in mathematics soon by Burkhardt (1964) when solving a practical situation (Fig. 1). This cycle focuses more on techniques for modeling, which illustrates cyclicality with two loops of the simplification and improvement, which are also the way scientists apply to build models.

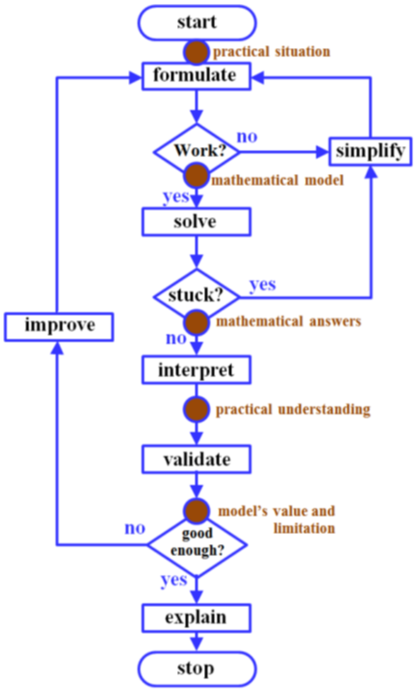

Fig.1 Modelling cycle (Burkhardt, 1964)

The modeling cycle building the scientific knowledge was proposed by Gentner (1983) (Fig. 2). In comparison to the cycle of Burkhardt, this cycle is less focused on the technique of creating models, but emphasizes on the goal of modeling and the test loops for models are built through two loops with thought experiments and empirical tests.

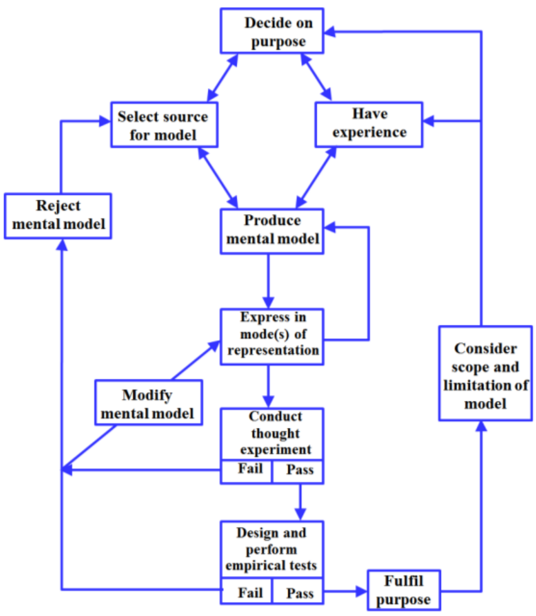

Fig.2 Modelling cycle (Gentner, 1983)

Hennes (2001) proposed a modeling cycle in mathematics teaching, which has many similarities with the teaching cycle of solving problems in science education (Fig. 3)

The modeling cycles in teaching mathematics today, that many education researchers mentioned, are proposed by Blum (2005) (Fig. 4). The two scopes which obviously are presented in this model are the mathematical world and real world. It can be seen that the model's cycle consists of 6 stations: Real situation, Mental model, Real model, Mathematical model, Mathematical results and Real results. Lines connecting stations represent activities during the modeling process, they are called Modelling paths. There are 7 Modelling paths: 1. Constructing; 2. Simplifying; 3. Mathematizing; 4. Working mathematically; 5. Interpreting; 6. Validating; 7. Exposing.

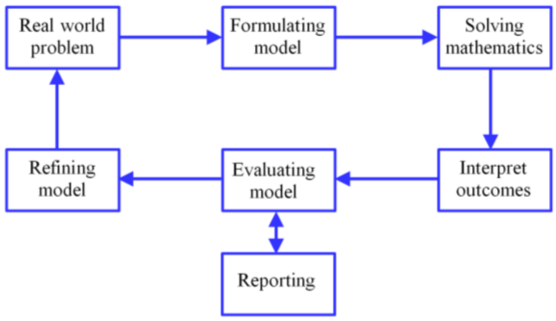

Fig.3 Modelling cycle (Hennes, 2001)

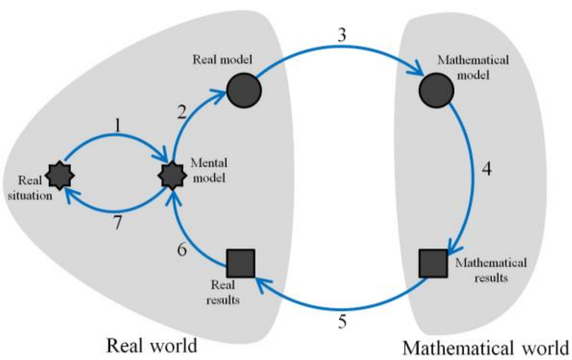

Fig.4 Modelling cycle (Blum, 2005)

The example "The Sugarloaf task" (Blum, 2007) has shown that this modeling cycle is described briefly, clearly and in accordance with applying mathematical models to explain specific problems in the real world. However, the role of the modeling cycle in teaching mathematics is currently mainly stopped at the level of helping students apply mathematical models into real world, which is still not the level, which helps students building a new mathematical model (Stillman, 2017). However, this modelling cycle applied in science education has a better role, not only to apply knowledge into practice but also to build and form new models (see section 1.2).

The modeling cycle should also be interpreted as a structural model of the modeling process, rather than an order of time of action steps (Prediger, 2010). Some studies have documented the action steps taken by students during modeling. It`s called "individual modeling path" (Ferri, 2007). The results of the study also indicate that the student's individual modelling paths generally differ from order of modeling paths in theoretical modelling cycle (Greefrath, 2017).

2. Applications of ICT in teaching physics

ICT with the leading role of the personal computer, handheld mobile devices, the internet system and the achievements of technological progress has a lot of support for teaching. The application of ICT in teaching physics can be cataloged into 5 functions:

- Describing: ICT supports the recording of images and videos of phenomena occurring in nature. Presentation of text, image and video data dynamically (e.g. software: PowerPoint, Windows Media Player ...)
- Connection: With internet network support, interactive information can be done anytime, anywhere. Students can observe a related phenomenon or review related knowledge at any learning stage (e.g. software: Internet Explorer, Chrome ...).
- Simulation and modeling: Computers can quickly perform a series of calculations on the data provided. Therefore, the time to calculate the parameters of the evolutionary state of real physical process over time can be quickly completed. However, the computer is just a machine that performs calculations. The command of the calculation steps must be controlled by humans through algorithm settings, that are concretized through computer programming language symbols. In order to be able to set up the algorithm, it is necessary to understand the dominant rule of phenomena over time (e.g. software: Stella II, Mathematica, Coach 7, Excel ...).
- Experimental data collection and processing: The sensor will convert the physical parameters into electrical signals. This electrical signal will go through an AD (Analog to Digital) adapter to standardize into a digital signal, and then it be connected to the computer to be recorded in memory, and be called data. These data will be processed according to the simulation or modeling scenarios (e.g.: Arduino and Inventor, Cassy and Cobra ...)
- Control: The data is obtained from a simulation to control an external device, it will be sent to the DA converter (Digital to Analog), in which it is converted into electrical signals. The electrical signals will be amplified and connected to an electrical device (e.g. an electric motor) to act as desired by the simulation (e.g. Arduino and Inventor, Cassy and Cobra ...).

3. Proposing a cycle of modeling mathematics and physics integration teaching with ICT support

Blum's modeling cycle (see section 1.3) has been widely applied in research of teaching mathematics. However, the proposal of a mathematical and physical integrated modeling cycle with the support of ICT has not been taken account by education researchers.

The supported mathematics learning cycle of ICT is briefly described in Fig. 5. In this model, there is a clear distinction between mathematical model and computer model (Greefrath, 2011a). However, as example 4.2 below indicates, that math and computer model without the presence of physics will lead to mistakes. Therefore, the computer model should not be separated from the mathematical and physical model. In another research of Greefrath (2011), the ICT action are not directly added into modeling cycle in learning mathematics, instead it describes each stage of the modeling cycle with integration of ICT support.

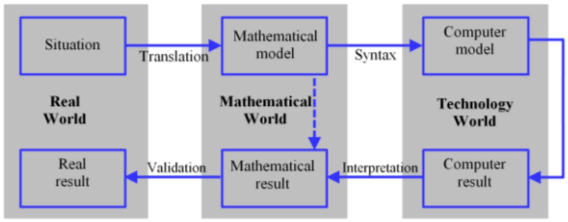

Fig.5 Modelling cycle (Greefrath, 2011a)

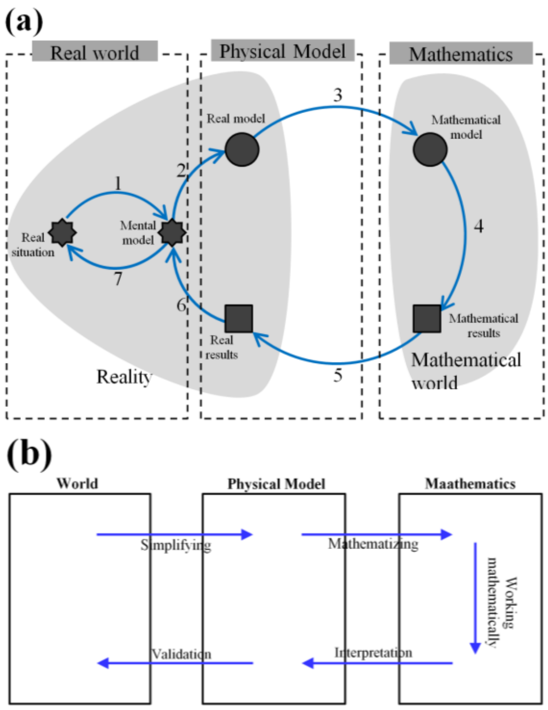

Fig. 6 Modelling cycle (Uhden, 2012)

When applying Blum's modeling cycle in physics learning, the modeling cycle can be divided into three independent scopes: Real model, Physical model and Mathematics (Fig. 6). However, it is in the scope of physics that structures of mathematical thinking exist (Uhden, 2012), and in mathematical reasoning steps, there must also be the guidance of physical thinking (see section 4.2). Therefore, in the process of modeling having mathematical and physical integration with the support of ICT, it is not necessary to clearly distinguish the scope of physics and the scope of mathematics.

From the consideration of the influence of Blum`s modelling cycle, the considerations of ICT-supported mathematical modeling cycles and considering the study of embedding the mathematical modeling cycle in physics learning process, a new modeling cycle is proposed. This modelling cycle was modified from the Blum's modelling cycle and allows describing steps to build or apply integration knowledge of physics and mathematics with the support of ICT (Fig. 7).

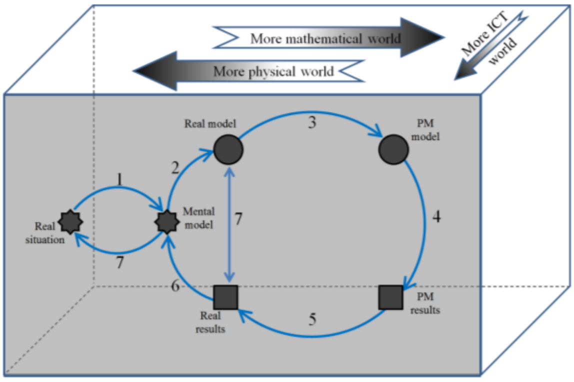

Fig.7 New modelling cycle concerning mathematics, physics and ICT integration

The modeling cycle is presented in 3D space, corresponding to physics, mathematics and ICT. The modelling cycle still preserves the number of stations as Blum's modelling cycle, but the content of stations 4 and 5 is renamed Physical and mathematical model (PM model) and Physical and mathematical results (PM result). In addition, the model also adds a 6b path module, which means testing by experiment (Experimenting). Actions to go from one station to the next can be supported by ICT, allowing achieving a higher efficiency in learning. The scope of mathematics and physics overlap completely on each other, but pointing on the right side is more related to the mathematical world, pointing to the left will be more about the physical world. The third dimension, the depth of model shows how deep the level of ICT support is.

4. Illustration of the new modelling cycle through examples

To illustrate the application of the modeling cycle proposed above, the following 4 examples of different problematic situations in traditional swinging folk games in Vietnam are presented (Fig. 8). Stations and modeling paths descriptions for each station corresponding to the modeling cycle will be specified. In addition, the level of ICT support and ICT role in modeling will be analyzed.

4.1. The first example

The situation of swinging games in Vietnam: The observer uses a clock to measure the time between two consecutive times that the gamers reach to the maximum height on the left: 5s. The question is to determine the height of the bamboo frame. The following will describe 6 stations and 7 modeling paths corresponding to the proposed modeling cycle:

Station 1. Real situation: Reconciliation with the modelling cycle, the Real situation station is the game observation experience. (Fig. 8). The time reaching the maximum height sequentially almost remains constant.

Station 2. Mental model: After understanding the real situation, the initial visualization of the situation will form a Mental model (corresponding to modeling path 1: Constructing). Each individual will have a unique mental model (Fig. 9).


Fig.8 Real situation

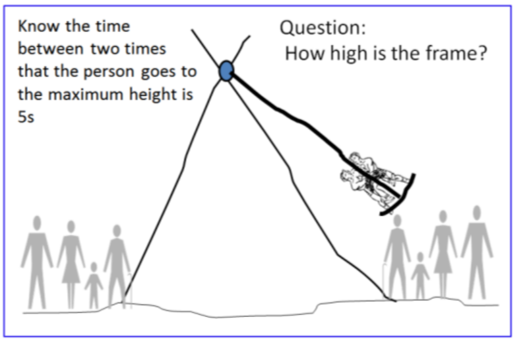

Fig.9 Mental model

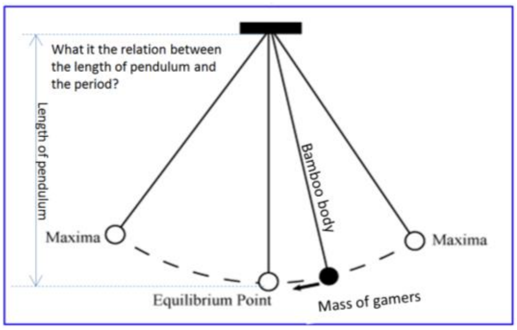

Fig.10 Real model

Station 3. Real model: From the mental model some key elements were refined, and then to be simplified (corresponding to modeling path 2: Simplifying). Consider this game layout to be simplified, similar to the experimental layout of a single pendulum. From that point on, the diagram was designed to illustrates that experimental device with notes about objects related to the real situation (Fig. 10).

Station 4. Physical and mathematical model: The physical objects with its characteristics and the relationships from the real model will be represented mathematically and physically (corresponding to modeling path 3: mathematization). At this station, the physical models and mathematical models were selected to represent the characteristics of the real model (Fig. 11)

Station 5. Physical and mathematical results: By applying mathematical laws the physical and mathematical results will be drawn (corresponding to modeling path 4: Working mathematically) (Fig. 12).

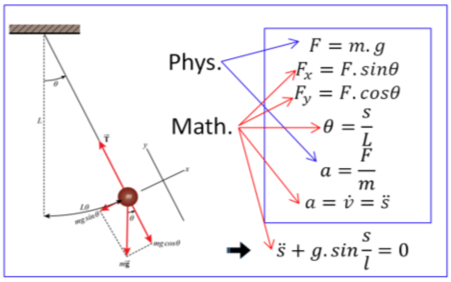

Fig.11 Physical and mathematical model

$$\ddot{s} + g.\frac{s}{l} \cong 0$$

$$\Downarrow$$

$$s = S_o.\sin(\frac{2.\pi.t}{T} + \varphi_o)$$

$$T = 2.\pi\sqrt{\frac{L}{g}}$$

Fig.12 Physical and mathematical results

T is constant when L and g are constant. T depends on L according to the following formula:

$$L = g.\frac{T^2}{4.\pi^2}$$

After replace the data from real situation, the height L is

$$L = 6{,}2m$$

Fig.13 Real results

Station 6. Real results: From physical and mathematical results, the actual results will be explained (corresponding to path 5: Interpreting) (Fig. 13).

With this result, a single pendulum experiment must be carried out to verify the relationship between the pendulum length and the oscillation period (corresponding to path 6b: Experimenting). The test results confirmed the results. Therefore, the real result returned to the mental model, which help envision the height of the bamboo frame to be about 6.2m, which is approximately 4 times the height of the observer, which is likely to match the original image (similar Applying path 6: Validating). From this, it is possible to explain how to calculate the height of a vibrating object in practice, which has a single pendulum-like vibration (corresponding to modeling path 7: Exposing).

4.2. The second example

The initial situation and the goal are the same as in the first example (4.1.) However, from the 4th station to the 5th station we will not use the mathematical working, but instead ICT will be applied, namely the computer modelling (or computer simulation) function. First of all, we need to set up a computer modeling cycle. This process is essentially a discrete process of continuous physics. The usual way to do this process is to divide the process continuously into a lot of (n) differential processes that have small changes (Δt) in succession. We consider, that in the differential process the simplest physical states are evolving to facilitate calculations, in particular in this example, during small Δt, the acceleration, force, velocity and position are assumed unchanged. Moreover, between differential processes, the quantity of accelerations, forces, velocities and displacement will vary in quantity corresponding to the small time-varying Δt. Specifically, the additional quantity of displacement is Δs=v. Δt, the additional quantity of velocity is Δv=a. Δt, the force and acceleration will have new values depending on the position. Thus, if the initial position and velocity of the object were given, we will calculate to find out all quantities of position, velocity, force and acceleration of all n differential processes. If the real continuous process is divided into a lot of n differential processes (for example, n = 1000), then manual calculations will be very time consuming. However, if the computer program is used, all series calculations will take place very quickly.

An algorithm describing the calculation process as shown above is represented in Fig. 14. The algorithm will guide the computer to perform the calculation of every quantity according to every differential process. In order to let the computer understand this algorithm, it is necessary to program in the grammatical structure (write code) corresponding to the given software. The computer, after reading the programming code, quickly executes a series of calculations based on the algorithm, thus finding out all series of data of physical quantity for every differential processes. Computer programming today has been simplified, maybe just drag and drop icons on the screen (e.g. software: Stella II, Mathematica, Coach 7, Excel ...). By the use of software with support for modeling and simulation, the results can quickly be achieved (Fig. 15). This result will enable mathematics and physics to draw results that are equivalents the station 5 at example 4.1 (Fig.12).

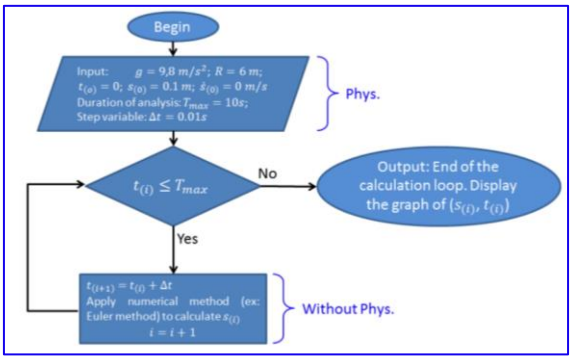

Fig.14 Algorithm to calculate at every differential process

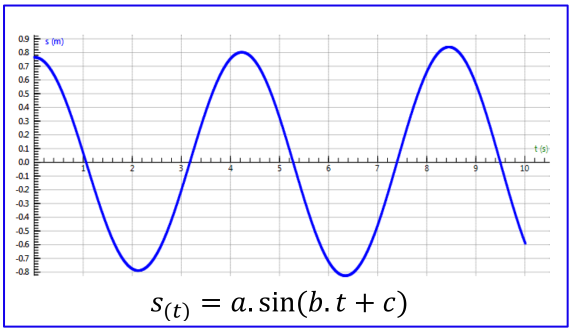

Fig.15 ICT result

Through the above analysis, it is required to create an algorithm to perform computer-based calculations. So it must be including the physical thinking, not merely the math and a technique to create algorithm. For example, in the above case, if the actual evolution of the physical process is without consideration, we don't have the criteria to choose the value of dt. So it can be chosen Δt = 0.1s. Consequence, the result of the oscillation process is a false (Fig. 16). However, mathematics and ICT cannot know right away that this is the wrong result even though the calculation steps are logical. Therefore, activities in the modeling process must be activities that integrate mathematical, physical, and ICT thinking.

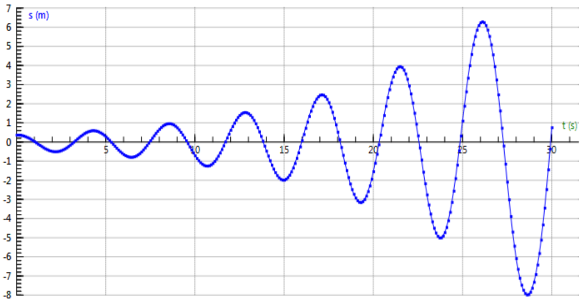

Fig.16 One wrong ICT result

4.3. The third example

In this example will show that the application of ICT is the only way to solve the problem. Return to the swing game at 4.1: After performing modeling, the period of swing must be constant. However, while the swinging, if the players make the swing amplitude reach the higher position, then a greater period of swing is observed. In the case, that the player makes the bamboo body move up, and the angle between bamboo body and the vertical is greater than 90 degrees, the period of swing will increase very much. The period in this case can be recorded up to 16s (Fig. 17). Therefore, if applying the result from the modeling cycle as in example 4.1, the height of the bamboo frame is too large: 64m. A result far from reality.

However, if using the modeling cycle with ICT support in calculating among series of differential processes, we still get reasonable results for the height of bamboo frame: about 6.2m. In this case, the player's movement will not follow the harmonic rule (Fig. 17)

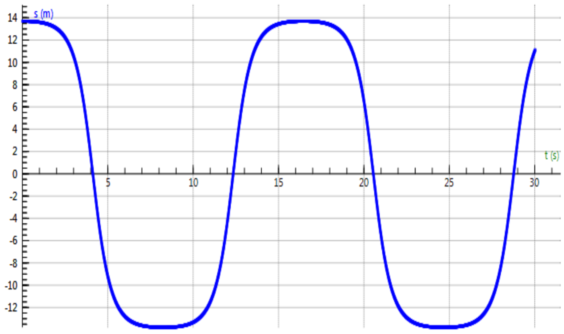

Fig.17 ICT result for special case of Vietnamese swing game

4.4. Example 4

This example will continue to show ICT support in more difficult situations. ICT not only supports the modeling cycle at the calculating stage (Working mathematical) but also supports the experimenting phase with data collection and device control functions.

Going back to example 1, a swinging game due to friction, then the swing will have to be damping. However, the players can still create sustained swing. So what should a player do to maintain periodic oscillation?

From the actual observation, swingers do not apply external forces to the system. However, they will shrug and stoop their body at different level to maintain oscillation. When shrugging or stooping, player will change the centroid. It leads to change the relative position of center of gravity, therefore change the gravitational potential energy. So PM model (Station 4) will be: by changing the the centroid the players will provide the gravitational potential energy for the system to sustain the swing. Station 5.

(PM result): The results of mathematical arguments and physical models are: When the players change the position of center of gravity from orbit a to orbit b (Fig. 18), the gravitational potential energy will decrease, so the kinetic energy must increase. After that, the players change the position of Centre of gravity from orbit b back to orbit b and accompany by the obtained kinetic energy. This added energy will compensate for energy wastage due to friction, thus maintaining the swing.

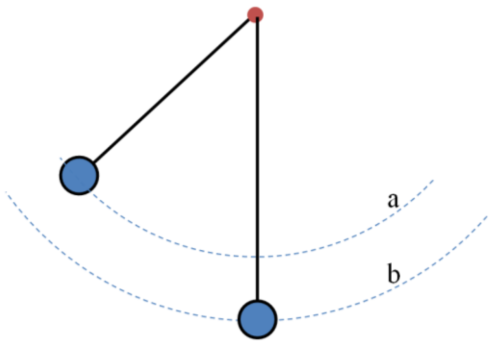

Fig.18 PM results explaining how to maintain the swing

Station 6 (Real result): So when the players reach the highest position they should to stand upright to remain high position of Centre of gravity. When the players reaches its lowest position, it is necessary to shrug and stoop to lower the center of gravity to obtain a higher velocity, so the kinetic energy will be added. Therefore providing more energy to maintain the swing.

To test this will require a single pendulum experiment, with the studied system including a servo and a heavy metal cube attached to the arm of servo. The servo will lift the heavy metal ball when the pendulum is reaching the highest point on the left side and to the highest point on the right side, and the servo will lower the heavy metal ball when the pendulum is moving to the lowest position. Servo control in such a way can be done with ICT with Arduino support. To actualize the process of lifting and lowering the heavy metal ball of the servo according to the pendulum oscillation state, an algorithm must be designed, and this also lead to programming code corresponding to that algorithm, too. Experimental results show that the pendulum having servo lifting and lowering metal ball can sustain the oscillation. This means, that the proposed model is reasonable, and the result of modeling (Real result) will describe the technical guide for swing game players.

5. Conclusion

The integrated mathematical and physical learning process is carried out harmoniously when applying modeling from mathematical learning to physical learning with the support of ICT. This modeling process will make opportunities for students to inquiry and find new physical knowledge and to apply mathematical knowledge to practice. This combination also makes mathematical thinking and physical thinking harmoniously integrated.

The modeling cycle has many similarities with the knowledge discovery cycle. Therefore, the application of mathematical models into physics can be understood as a cycle of building new physical knowledge. Teaching new knowledge can follow the process of modeling. However, it should be noted that the modelling cycle indicate the structure of actions rather than the order by time of actions in the modelling cycle.

The examples presented in section 4 treated the same topic, but the degree of difficulty in situations and the ways of dealing are also different, which helps to understand and apply the modeling cycle in teaching integrating mathematics and physics more clearly. In addition, these examples also allow teachers to organize students to implement the following diverse modeling cycles to suit different learners' competence. Applying this method will also bring many positive learning benefits (Lamb, 2017).

ICT in this era can support at all stages of the modeling process. However, supporting ICT requires the integration of many mathematical and physical knowledge, in which computer modeling must play the main role. When students achieve modelling cycles, they could develop modeling competence, but this is also a process for students who are struggling. Learners tend to try out an existing model, but the ability to build computer models and building steps to calculating are still limited (Sins, 2005). Improving the competence of analyzing the physical process to facilitate the loops calculation of computers, how teacher can organization students' learning as well as the impact on student should continue to be researched.

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