

A Computer Simulation of Cosmic Inflation

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Abstract

From the Cosmic Microwave Background CMB, the flatness problem and the horizon problem arose. An extraordinarily increase of distances in the early universe, the Cosmic Inflation, was proposed as a possible solution, whereby suggested mechanisms for such an increase have been criticized (Steinhard, 2011). We apply a theory that explains the Cosmic Inflation by an extended Friedmann-Lemaitre model combined with an energy term (Carmesin, 2017). We investigate various questions by performing computer simulations. We observe a sequence of phase transitions that cause an extraordinarily fast increase in distances. Our findings are in excellent quantitative agreement with observations of the CMB. Thereby the theory depends only on first principles and the fundamental constants G , c and h and we apply no fit in particular. We present the development of the project in the framework of a Jugend forscht club.

1. Introduction

From the Cosmic Microwave Background CMB, the flatness problem and the horizon problem arose. To describe the expansion of the universe, a Friedmann-Lemaitre model has been frequently used until now (see Karttunen 2007). The Friedmann-Lemaitre differential equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \cdot (\rho_M + \rho_V) \quad \{1\}$$

does not solve the two problems arising from the Cosmic Microwave Background. One additional problem, which is not solved by the Friedmann-Lemaitre equation is the singularity problem (see Kiefer 2008), which considers a singularity as non-physical. In the Friedmann-Lemaitre equation ρ_M is the matter density and ρ_V is the vacuum density. The vacuum density is constant. The solution of the Friedmann-Lemaitre model provides the evolution of the scale factor $a(t)$.

2. Problems

The singularity problem is visible in the solved Friedmann-Lemaitre equation, where the density tends to infinity, when $a(t)$ and t tends to zero (see figure 1).

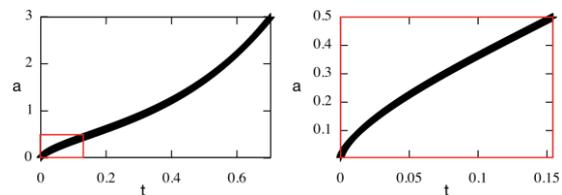


Figure 1: Symbolic evolution of the Friedmann-Lemaitre equation, numerically solved. With $G = 1$, $\rho_V = 1$ and $m = 1$.

The horizon problem and the flatness problem arise from the CMB (see figure 2).

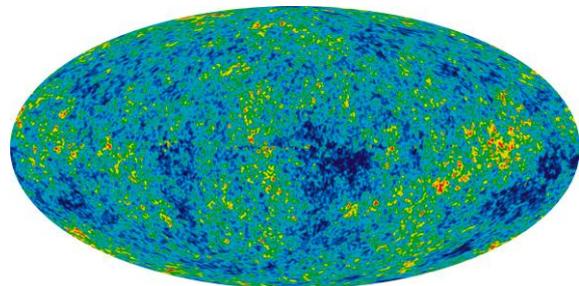


Figure 2: Cosmic Microwave Background (figure: NASA WMAP Science Team, 9 year WMAP image of background radiation).

3. The horizon problem

The horizon problem arises from the temperature fluctuations which amount to $\frac{\Delta T}{T} = 0.000024$. So the

full solid angle 4π covered by the Cosmic Microwave Background has a homogenous temperature distribution. The full solid angle is so large that on the basis of the Friedmann-Lemaitre equation it would be causally disconnected and it would not be possible for radiation to distribute the energy homogeneously in the time since the big bang.

4. The flatness problem

The second problem arising from the CMB is the flatness problem. The space-time curvature is measured and indicates a flat space on a large scale. Modelled with the Friedmann-Lemaitre equation it would have been even more flat in the early universe which is highly unlikely.

5. Possible solution for the problems

To solve the three problems a rapid increase in distances in the early universe, the Cosmic Inflation, was proposed by Allan Guth in 1981 (see Guth 1981). In many approaches to the problems an inflaton field is used, which requires very sensitive fit parameters to fit to observations. In this paper, we propose a new model without any fit parameters, an extended Friedmann-Lemaitre equation, to describe the Cosmic Inflation and solve these problems.

6. The model

The standard Friedmann-Lemaitre equation can be derived from a spherical model with a scale factor a , a probing mass m and the density ρ of the sphere.

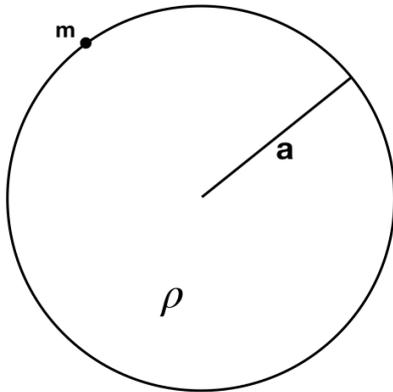


Figure 3: Model from which the Friedmann-Lemaitre equation can be derived.

From this spherical model, we can derive the Friedmann-Lemaitre equation (see Carmesin 2017, Carmesin 2018 a,d):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \cdot (\rho_M + \rho_V) \quad \{2\}$$

We extend the model by adding a radius for our probing mass in order to describe the density of the probing mass, which was not possible in the old model based on a point-like mass.

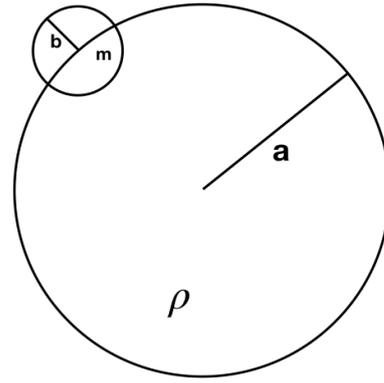


Figure 4: Model from which the extended Friedmann-Lemaitre equation can be derived.

In order to obtain the corresponding dynamics, we derive the quantum physical expectation values $\langle \dot{a} \rangle$ and $\langle a \rangle$ and denote these by \dot{a} and a in the following (for details see Carmesin 2018 a,d). In the resulting extended Friedmann-Lemaitre equation we introduce the scaled density $\tilde{\rho} = \frac{\rho}{\rho_{PD}}$ which is the matter density divided by the maximal density, the Planck density ρ_{PD} . From this new spherical model (see figure 4), we get the following extended Friedmann-Lemaitre equation for three dimensions (see equation {3}). Hereby we introduce the Planck time t_P .

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{t_P^2} \cdot \tilde{\rho} \cdot \left[1 - 6 \cdot \tilde{\rho}^{\frac{1}{2}}\right] \quad \{3\}$$

Moreover we generalize this model to spatial dimensions $D \geq 3$ and obtain the following extended Friedmann-Lemaitre equation (see Carmesin 2018 a,d, see {4})

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{t_P^2} \cdot \tilde{\rho} \cdot \left[1 - \sqrt{2^{D-2} \cdot (D-1) \cdot D^2} \cdot \tilde{\rho}^{\frac{(D-1)^2}{2(D+1)}}\right] \quad \{4\}$$

with the corresponding scaled energy term (see Carmesin, 2018 a,d, see {5}):

$$E_D = -\tilde{\rho}^{\frac{D-1}{D+1}} + \sqrt{2^{D-2} \cdot (D-1) \cdot D^2} \cdot \tilde{\rho}^{\frac{D-1}{2}} \quad \{5\}$$

Thereby the scaled energy term E_D is the quantum physical expectation value $\langle E \rangle$ at the ground state divided by $m \cdot c^2$.

7. Dimensional transitions

The scaled energy E_D is calculated for each scaled density $\tilde{\rho}$ and for any Dimension $D \geq 3$ (see figure 5).

$\tilde{\rho}$	0.009	
D	E_D	
3	-0.0408683298050514	
4	-0.047399707332933	
5	-0.0409764611381778	
6	-0.0341612190261753	
7	-0.0291494031505907	
8	-0.0256245858948968	
9	-0.0230865194541782	
10	-0.0211929929871053	
11	-0.0197333613375166	
12	-0.0185769436727649	

Figure 5: Scaled energy E_D as a function of the spatial dimension D at the scaled density $\tilde{\rho} = 0.009$.

From the table in figure 5 we get the following graph.

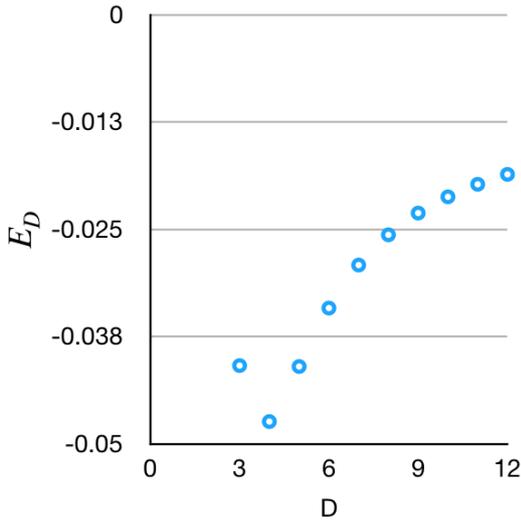


Figure 6: Scaled energy as a function of the spatial dimension D at the scaled density $\tilde{\rho} = 0.009$. Here the ground state energy is at $D = 4$.

In order to apply the D -dimensional version of the extended Friedmann-Lemaitre equation (see equation {4}), we calculate a critical density $\tilde{\rho}_{cD}$ at which there occurs a dimensional transition of the ground state from $D + 1$ to D for each dimension. For it we minimize the scaled energy term (see equation {5}). By applying the variational principle, we obtain the following values for the critical scaled densities (see figure 7).

Dimension D	Scaled critical density $\tilde{\rho}_{cD}$
3	0.006560789873286
4	0.0154690495420951
5	0.0270430431866556
6	0.0398603342266119
7	0.0530506248138605
8	0.0661239958458446

Figure 7: Critical scaled densities for dimensional transitions.

8. Solving the extended Friedmann-Lemaitre equation

The extended Friedmann-Lemaitre equation can be solved with numerical integration. For it we used the Runge-Kutta method of fourth order.

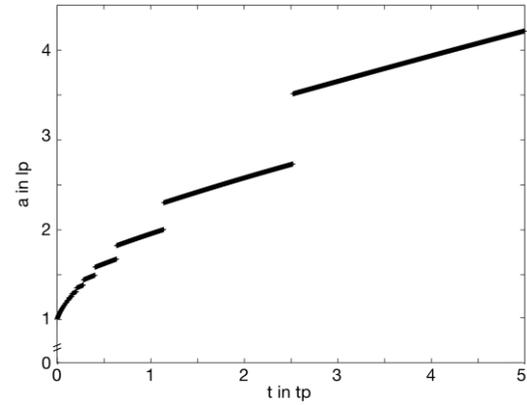


Figure 8: Solved extended Friedmann-Lemaitre equation: Discontinuities arise at dimensional transitions.

The dimensional transitions take place where the graph is not continuous (see figure 8). The graph in figure 8 shows symbolically the dimensional transitions with the corresponding increase in the scale factor. The exact values can be found in *Model for the Dynamics of Space* (see Carmesin 2018 a,d). Our critical density $\tilde{\rho}_{c3}$ can be utilized in order to calculate the observed density of ρ_M of the universe. Our result is in excellent accordance with observations. Thereby no fit must be applied (see Carmesin 2017, see Carmesin 2018 a,d).

9. Summary

We solve the extended Friedmann-Lemaitre equation generalized for spatial dimensions $D \geq 3$ numerically (see section 8). So we obtain the scaling radius a as a function of the time t including dimensional transitions at critical densities $\tilde{\rho}_{cD}$ (see section 7). Based on this solution the singularity problem and the flatness problem can be solved when the durations of the dimensional transitions are calculated with help of Fermi's golden rule (see Carmesin 2018 a,d). Furthermore these durations show in full detail how the singularity problem is solved by the dimensional transitions (see Carmesin 2018 a,d).

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