How Old is the Universe?
- A Teaching Unit Using the Novel Applet Spectrarium -

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Abstract
We present a teaching unit that offers the students the opportunity to discover the age of the universe by hands on model experiments also with applets. We present a novel software, worksheets and outlines of lessons. We present experiences with students and a teacher training.

1. Introduction

We present a teaching unit in which the students discover the age of the universe by simulated observations using applets. So the students can form hypotheses, test them and achieve a high efficiency of learning [1]. We used the well known planetarium-software Stellarium [2] and a software simulating astronomical spectroscopic observations. We developed the latter software and called it Spectrarium. The teaching unit is presented in three versions, a conceptual level, an expert level and a mathematical level. The units have been tested with students in Germany and in a teacher training in Lisbon. We present the teaching unit including corresponding work sheets and report about our experiences in lessons and teacher training. The software Spectrarium and a used "zoom animation" can be requested via E-Mail.

2. Conceptual Level

The unit for the conceptual level consists of five lessons. It has been tested in an astronomy club with students from classes five to 12. The outlines of the lessons including working sheets are as follows:

Lesson 1: Investigation of the Large Magellanic Cloud LMC

Goals: The students can find the LMC with Stellarium, describe the location of the LMC at the celestial sphere as well as essential contents of the LMC.

Time	Didactical Comments	Methodical Comments	Social form
8	Introduction: Magellan	Story of his sailing around the world, his discovery of the strait of Magellan, his description of the LMC	Discussion
10	Development of the question	Question of the lesson	,,
50	Development of results: exploration	Stellarium, zoom-animation	Small Groups
70	Formulation of results: see below	Formulation and animation in the plenum	Student-presentation
90	Consolidation: exploration of further galaxies	Stellarium	Small Groups

Intended Blackboard Figure:
Where is the Large Magellanic Cloud at the Celestial Sphere and What Can We Find in It?
Ideas: Southern Hemisphere √
LMC contains: stars, star clusters
Results: The LMC is in the constellation Dorado. It contains globular star clusters and open star clusters. It is a galaxy.

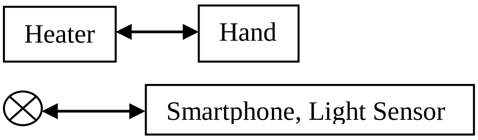
Lesson 2: Discovery and use of the decrease of radiation with distance

Goals: The students can explain the decrease of the intensity of a radiation source and its use for measurements of astronomical distances.

Time	Didactical Comments	Methodical Comments	Form
8	Introduction: Examples of distances see below)	Teacher instructs, students give a resume	Instruction
10	Development of the question	Question of the lesson	Discussion
20	Hypotheses: see below	Conjecture	,,
30	Experiment: Plan see below	Planning	,,
60	Development of results: performing the experiment	Some groups perform experiment, some develop the idea of the use of the decrease	Small Groups
75	Formulation of results: see below	Formulation and demonstration in the plenum	Pupil-Presentation
90	Consolidation:		Small

	Creative experimentation with more radiation sources		Groups

Intended Blackboard Figure
Information:
Distance to America: 5000 km
Distance to the moon: 380 000 km
Distance to the sun: 150 000 000 km
How Can the Distance of a Galaxy be Measured?
Hypothesis: we get only light, we use light
The light brightness decreases with the distance.
Design of model-experiments:



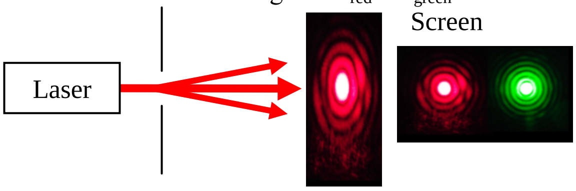
Results: The intensity of a radiation source decreases with the distance.
The galaxy is a radiation source.
The intensity of the light of the galaxy decreases with the distance.
Conversely, the astronomers measure the intensity of the light of the galaxy and therefrom calculate the distance.
Example: The galaxy M66 has the distance 280 Zm = $280 \cdot 10^{21}$ m.
If each human travels the distance to the sun, then they travel a distance of 1 Zm altogether.

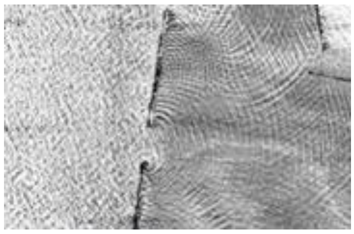
Lesson 3: Discovery and use of the *Wave nature of Light WNL*

Goals: The students can illustrate the WNL.

Time	Didactical Comments	Methodical Comments	Social form
5	Introduction: Question, see below	Teacher presents the question	Discussion
10	Ideas: see below	Students suggest	,,
20	Instruction: Diffraction experiment, Photo of watewaves,	Students describe the diffraction pattern and the photo of waterwaves.	,,
40	Development of results: Analogy	Students discover common property of waterwaves and light, see below	Small Groups
75	Formulation of results: see below, hand spectrograph if available	Formulation and illustration in the plenum	Student-presentation
90	Consolidation: creative experiments with waterwaves		Small Groups

Intended Blackboard Figure
How can We Measure Properties of the Galaxy?
Ideas: no probes → analysis of light
An experiment illustrates: Light propagates like a wave and has a wavelength λ. $\lambda_{red} > \lambda_{green}$





Results:
Light spreads behind a hole.
Waterwaves spread behind a hole.
Light has properties of waves.
The wavelength corresponds to the colour.
The wavelength can be observed with a so called spectrograph.
Perhaps the wave property of light gives more information about a galaxy.

Lesson 4: Discovery and use of the redshift

Goals: The students can explain the redshift and its use for the determination of velocities.

Time	Didactical Comments	Methodical Comments	Form
8	Introduction: M66 in Stellarium, increase of wavelength	Students suggest measurements of D and spectrum	Discussion
10	Development of the question	Question of the lesson	Discussion
15	Hypotheses: see below	Conjecture	,,
50	Development of results: performing the control experiment	Students perform experiment	Small Groups
55	Formulation of results: see below	Formulation and animation in the plenum	Student-presentation
90	Consolidation: Correspondence of wavelength increase and velocity	Students work with worksheet or observe ducks in a lake	Small Groups

Intended Blackboard Figure
Why is the Wavelength Increased?
Hypothesis: Motion changes the wavelength
Control experiment:

- We move a toy duck in water
- Behind the duck the wavelength is increased
- In front of the duck the wavelength is decreased

Result: The wavelength of M66 is increased, because M66 moves away from us.
Information: From the increase of the wavelength, the velocity v of the galaxy can be determined with help of a spectrograph. The velocity is v = 20 Zm/Gy.
1 Gy = 1 billion years → ½ number of Europeans.

Worksheet, Astronomy Club, Dr. Carmesin 2013

Wavelength of a Moving Source

Exercise 1: Describe the wavelengths of the swimming baby duck (Photo from MINT Zirkel, September/October 2013, p. 18, hoc.).

Exercise 2: Find out what you can tell about the velocity of the duck by observing the wavelength.

Lesson 5: Discovery of the Equality of Start Times EST and Measurement of the Age of the Universe

Goals: The students can explain their discovery of the EST and calculate the age τ of the universe.

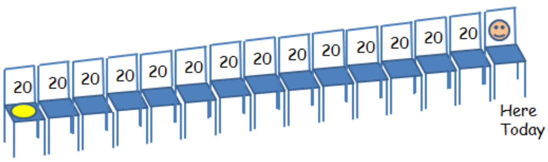
Time	Didactical Comments	Methodical Comments	Social form
8	Introduction: D, v, see below	Pupils explain	Discussion
10	Development of the question	Main question of the lesson	Discussion
20	Model experiment: see below	Plan	,,
30	Development of result: τ	Calculation	Small Groups
35	Formulation of result: see below	Formulation	Pupil-presentation
40	Development of the 2. question	2. question	Discussion
60	Development of result: τ	Calculation, worksheet	Small Groups
65	Formulation of result: see below	Formulation	Pupil-presentation
80	Metacognition: see below	Interpretation, Reflexion	Discussion
90	Consolidation: Investigate the proportionality of D and v	Worksheet	Small Groups

Intended Blackboard Figure

M66: D = 280 Zm; v = 20 Zm/Gy

Where Has M66 Been Before?

Design of model experiment:



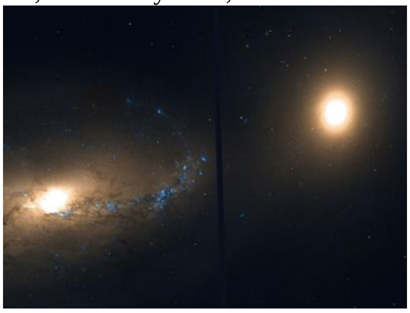
Result: 14 Gy ago M66 was here. The start time is 14 Gy.

When Have Other Galaxies Been Here?

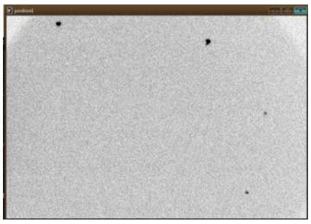
Result: All distant galaxies move away from us and started 14 Gy ago.

Interpretation: Everything began to expand 14 Gy ago. This is called the Big Bang. The age of the universe is 14 Gy.

Worksheet, Astronomy Club, Dr. Carmesin 2013



NGC3227: HST, D = 700 Zm; v = 50 Zm/Gy



NGC3516: Photo by pupils in Stade with 11'-telescope [3]. D = 1400 Zm; v = 100 Zm/Gy

Exercise: When have the galaxies been here?

3. Spectrarium

On the expert level the students simulate the astronomical spectroscopy of distant galaxies. For this purpose we developed the novel software "Spectrarium". The underlying concept is outlined in the following.

Overview of the concept of Spectrarium

First the spectral flux densities are determined for the radiation coming from the galaxy [4] and for the radiation originating from the light pollution [5]. These calculations are based on data from the literature.

Second the user can simulate his specific observation. Thereby the user can choose the diameter D of the telescope, the temperature T of the cooled camera, the exposure time t and the location with a light pollution described by a parameter L. For this purpose the incoming spectral flux densities of the galaxy and of the light pollution are transformed to the spectrum generated by the chosen parameters D, T, t and L. This transformation is realized by the software in two steps. In a first step we calculate the spectrum for the observatory in Stade equipped with the telescope Celestron C11, the spectrograph DSS7 form SBIG and the camera ST-402 from SBIG. In a second step we scale the spectrum according to the chosen parameters D, T, t and L.

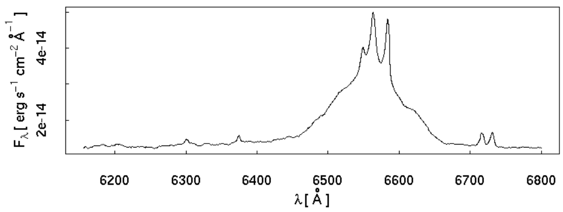


Fig.1: Spectrum of the galaxy NGC3516 [4].

Radiation from a galaxy

From the literature we get the incident spectral flux density F_λ coming from a galaxy. For instance, the H_α-radiation of the galaxy NGC3516 arrives at the earth with a spectral flux density of $F_\lambda = 5\cdot10^{-14} \frac{erg}{s\cdot cm^2}\cdot Å^{-1}$ or $F_\lambda = 5\cdot10^{-17} \frac{W}{m^2}\cdot Å^{-1}$ or $F_\lambda = 500 \frac{aW}{m^2\cdot nm}$ (see Fig. 1).

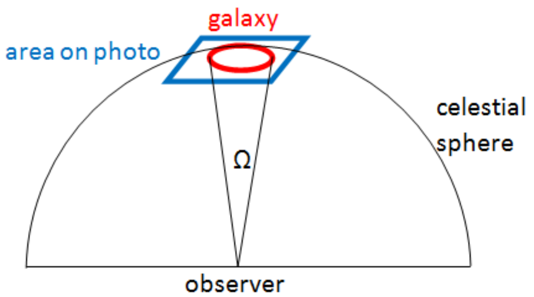


Fig.2: Construction for light entering the telescope

Spectral flux of the light pollution

From the literature we get the sky brightness or visual luminance L_V in candela per square meter [5]. For instance, the sky above the city of Stade emits light with a luminance of $L_V = 15 \cdot 10^{-4} \frac{cd}{m^2}$. At the bottom there arrives the same luminance. In order to get the illuminance E_V that enters the telescope, we multiply the luminance L_V with the solid angle Ω of the galaxy (see Fig. 2). This solid angle is the solid angle of the area presented on the photo (see Fig. 2) multiplied by the percentage of the area covered by the galaxy on the photo. For example, the telescope C11 with the camera ST-402 from SBIG produces a photo with a solid angle of $4.5\cdot10^{-6}$ rad². The galaxy covers an area of 0.1 % of this photo (see Fig. 3). Thus the solid angle of the galaxy is $4.5\cdot10^{-9}$ rad² and the illuminance E_V of the light pollution is $E_V = 6.675\cdot10^{-12} \frac{cd\cdot rad^2}{m^2}$ or $E_V = 6.675\cdot10^{-12} \frac{lm}{m^2}$. Next we convert this visual or physiological illuminance E_V to an energetic illuminance E_e. For this conversion we use the fact that the illuminance of the sun is $E_e = 800 \frac{W}{m^2}$ or $E_V = 100000 \frac{lm}{m^2}$. So $1 \frac{lm}{m^2}$ corresponds to $0{,}008 \frac{W}{m^2}$ and the energetic illuminance of the light pollution is $E_e = 54 \frac{fW}{m^2}$. This illuminance of the light pollution is an average over the visual wavelength–interval [400 nm; 800 nm] with $\Delta\lambda$ = 400 nm. So we obtain the spectral energetic illuminance alias the spectral flux density F_λ by dividing the illuminance by $\Delta\lambda$ = 400 nm. Thus we get $F_\lambda = 135 \frac{aW}{m^2\cdot nm}$.

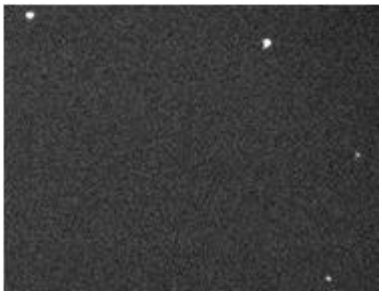


Fig.3: Photo of the galaxy NGC3516 (upper right) taken by pupils with an 11-inch-telescope.

Transformation to the spectrum for the observatory in Stade

We present the spectrum for the wavelengths 365 nm, 366 nm, 367 nm, … 709 nm. The opening of the telescope has a diameter of D_0 = 0,275 m. With our observatory we took a reference spectrum at a clear night from the sky in Stade (see Fig. 4) at a temperature of T_0 = 270 K with an exposure time of t_0 = 300 s. This spectrum exhibits a maximum of the light pollution at 1122 counts and λ = 544 nm originating from mercury in the street lamps. Moreover this spectrum shows a stochastic noise characterized by the standard deviation σ_0 = 97.

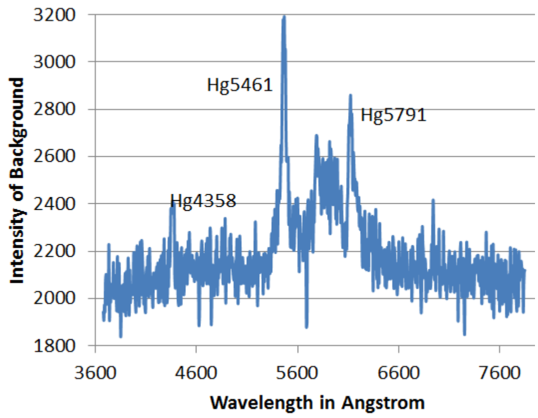


Fig.4: Reference spectrum

In order to produce a light-pollution-spectrum, we subtracted the noise from the reference spectrum and rescaled this spectrum so that the mean spectral flux is $F_\lambda = 135 \frac{aW}{m^2\cdot nm}$. As a result we obtained a maximum spectral flux of $1245 \frac{aW}{m^2\cdot nm}$ at 544 nm. So the standard deviation of the stochastic noise should be $\sigma = 1245\cdot97/1122 \frac{aW}{m^2\cdot nm}$ or $\sigma = 108 \frac{aW}{m^2\cdot nm}$. We generate such a noise-spectrum with the help of random numbers. Thereby we add ten random numbers for each wavelength and rescale appropriately. We take the galaxy-spectrum from the literature. Finally we obtain the displayed spectrum by adding the above three spectra. Shortly speaking, we form the sum displayed spectrum = galaxy-spectrum + noise-spectrum + light-pollution-spectrum.

Rescaling of the spectra

The user of the software may simulate the observation with a location with a pollution factor L, with a telescope with diameter D, with a temperature T and with an exposure time t. As a consequence the three spectra are scaled by the following factors:

- The galaxy-spectrum is obviously multiplied by the following galaxy-factor: $g_f = (D/D_0)2 \cdot t/t_0$
- The light-pollution-spectrum is obviously multiplied by the following pollution-factor: $p_f = (D/D_0)^2 \cdot t/t_0 \cdot L/L_0$
- The noise-spectrum is multiplied by a noise-factor of the following form[1]: $z_f = (t/t_0)^{0.5} \cdot \max(1; 2^{[(T-270)/6.3]})$. Thereby T is the temperature in K and we obtained the parameters of this formula from a series of experiments.

4. Expert Level

On the expert level the students can simulate and evaluate all observations. The unit has been tested with students in an astronomy club from classes six to 12. The unit consists of six lessons outlined as follows:

The first lesson is the same as on the conceptual level.

Lesson 2: Discovery and use of the *Inverse Square Law of Flux Density ISL*

Goals: The students can explain the Inverse Square Law of Flux Density and use it for calculations of astronomical distances.

Time	Didactical Comments	Methodical Comments	Form
8	Introduction: Information about α1 Centauri, see below	Teacher instructs, students give a resume	Instruction
10	Development of the question	Question of the lesson	Discussion
20	Hypotheses: see below	Conjecture	,,
30	Experiment: Plan see below	Planning	,,
60	Development of results: performing the experiment	Some groups perform experiment, some groups derive formula	Small Groups
75	Formulation of results: see below	Formulation and animation in the plenum	Pupil-Presentation
90	Consolidation: Calculation of fluxes and distances	Worksheet	Small Groups

Intended Blackboard Figure

Information:
α1 Centauri emits the same spectrum as the sun.
So α1 Centauri has the same physical properties as the sun.
So α1 Centauri emits the same power as the sun $P_{sun} = 3.85 \cdot 10^{26}$ W.
The sun appears brighter than α1 Centauri.
A modern measure of brightness is the power P per area A, the so called flux density F = P/A.

Why has α1 Centauri a smaller Flux Density than the Sun?

Hypothesis: light spreads in space.
The light brightness decreases with the distance.
Experiment design:

Results: Inverse Square Law of Light Brightness ISL:
The flux density F decreases with the distance D proportional to $1/D^2$.
The flux density F of a source with power P alias flux L at a distance D is $F=P/(4\pi D^2)$.
The flux density F of a source with an apparent brightness m is: $F = 1367\ W/m^2 \cdot 10^{-0.4\cdot(26.83+m)}$
Calculation of the distance of α1 Centauri:
$P = 3.86 \cdot 10^{26}$ W
$F = 25.6\ nW/m^2$
$D = (P/[4\pi F])^{0,5}$ = 34.6 Pm = 3.6 Ly (Stellarium: 4.39 Ly)
Calculation of the flux of β Centauri for comparison:
Stellarium: Apparent Brightness m = 0.55
$F = 1367\ W/m^2 \cdot 10^{-0.4\cdot(26.83+m)} = 15.3\ nW/m^2$

Worksheet, Astronomy Club, Dr. Carmesin 2013
Usual stars are spherical gas balls performing a nuclear reaction. As a consequence the colour of a star corresponds to its flux L or emitted radiation power P. This is conventionally expressed by the following classification:

Class	**Colour**	**Flux or power in multiples of the power of the sun, P in P_{sun}**
O	Blue	Larger 30000
B	Blue-white	25-30000
A	White	5-25
F	Yellow-white	1,5-5
G	Yellow	0,6-1,5
K	Orange	0,08-0,6
M	Red	Smaller 0,08

$P_{sun} = 3.85 \cdot 10^{26}$ W
This is the basis fort he spectroscopic distance measurement, illustrated for Sirius:

Exercise 1:Determine the apparent brightness m with Stellarium.
Exercise 2: Determine the spectral class with Stellarium.
Exercise 3: Determine the flux density F from the apparent brightness m.
Exercise 4: Determine the flux P approximately from the spectral class.
Exercise 5: Determine the distance D from F and m.

[1] The exponent 0.5 is a consequence oft he central limit theorem. The denominator 6.3 describes the fact that the noise is increased by a factor two if the temperature is increased by 6.3 K. The maximum describes the fact that the thermal noise dominates for temperatures above 270 K, while the read-out-noise dominates for temperatures below 270 K.

Intended solution:
Stellarium: m = -1.45 →
$F = 1367\ W/m^2 \cdot 10^{(-0.4\cdot(26.83+m))} = 96\ nW/m^2$
Stellarium: Spectral class A →
approximately we get $P = 15\ P_{sun}$
$D = (P/(4\pi\cdot F))^{0.5} = 69$ Pm = 7.2 Ly
(Stellarium: 8.6 Ly)

Lesson 3: Measurement of the distance of a galaxy with an 11' telescope

Goal: The students can simulate with Stellarium the measurement of the distance of a galaxy with an 11' telescope.

Time	Didactical Comments	Methodical Comments	Social form
8	Introduction: Locate NGC3516 in Stellarium	Simulation	Discussion
10	Development of the question	Main question of the lesson	Discussion
15	Ideas: see below	Conjecture	,,
20	Info 1: Measures of brightness	Instruction	Instruction
30	Development of result 1: F = 0.58 pW/m^2	Calculation	Small Groups
40	Formulation of result 1: see below	Formulation	Pupil-Presentation
45	Development of the question	2. question of the lesson	Discussion
60	Info 2: Galaxy Survey	Worksheet, exercise	Small Groups
70	Development of result 2: D = 1300 ZM	Calculation	Small Groups
75	Formulation of result 2: see below	Formulation	Pupil-Presentation
90	Consolidation: galaxies found in Stellarium	Calculation of distances	Small Groups

Intended Blackboard Figure

How Can We Measure the Distance D of the Galaxy NGC3516?

Ideas: We only get light from NGC3516 → we analyse light → we analyse the brightness
We use Stellarium: m = 11.6
Measures of brightness observed at the earth:
The magnitude m is an ancient unit. Example: Polaris observed at the earth: m = 1.95
Further Examples: Vega: m=0. (Mizar; Alcor) → (2.2; 3.95). ηUMi: m = 4.95. Just visible: m = 6
The flux density F has the modern unit W/m^2. Polaris observed at the earth: F = 4.2 nW/m^2
Conversion m to F: $F = 1367\ W/m^2 \cdot 10^{-0.4\cdot(26.83+m)}$
NGC3516: $F = 1367\ W/m^2 \cdot 10^{-0.4\cdot(26.83+11.6)} = 0.58\ pW/m^2$

What is the Radiation Power P alias Flux L of a Galaxy?

Galaxy survey (Yee, H. K. C. u. a.: The CNOC2 Field Galaxy Redshift Survey. I. The Survey and the catalogue for the Patch CNOC 0223+00. The Astrophysical Journal Supplement Series, 129, 475-492, 2000):

With P = 13 trillion YW = $13\cdot10^{36}$ W the distances have a mean error of 33.8%.
Exercise solution: $D = (P/[4\pi F])^{0,5} = (13\cdot10^{36}\ W/[4\pi\cdot180\ fW/m^2])^{0,5} = 2397$ Zm
Result 1: Using the typical galaxy flux $13\cdot10^{36}$ W, we get a mean distance error of 34 %.
NGC3516: $D = (P/[4\pi F])^{0,5} = (13\cdot10^{36}\ W/[4\pi\cdot0.58\ pW/m^2])^{0,5} = 1300$ Zm (Lit.: 1140 Zm)
Result 2: Using Stellarium, we measured the distance 1300 Zm.
Info: Using an 11 inch-telescope, pupils measured: m = 11.1; F = 0.92 pW/m^2 ; D = 1060 Zm

Worksheet, Astronomy Club, Dr. Carmesin 2013
Table:
Columns 1-3: Galaxy Survey
Column 4:
$D_{theoretical} = (P/(4\pi F))^{0,5}$ with:
P = 13 quadrillion YW or
P = $13\cdot10^{36}$ W
Column 5: $|D_{measured} - D_{theoretical}|\cdot100\%$
Exercise: Control $D_{theoretical}$ for Nr. 1.

			with	
			P = 13 quadrillion YW	
		$D_{measured}$	$D_{theoretical}$	Error
Nr.	F in fW/m^2	in Zm	in Zm	in %
1	180,07	3113	2397	23,0
2	5,82	32805	13327	59,4
3	4,50	11882	15170	27,7
4	3,79	19490	16511	15,3
5	2,95	34900	18724	46,3
6	2,61	31037	19915	35,8
7	2,20	37701	21670	42,5
8	1,96	46868	22973	51,0
9	1,85	35459	23625	33,4
10	1,64	32756	25099	23,4
11	1,45	40048	26722	33,3
12	1,46	66121	26616	59,7
13	1,21	31105	29281	5,9
14	1,17	49863	29722	40,4
15	1,11	44772	30500	31,9
16	1,02	35010	31912	8,8
17	0,95	51606	32973	36,1
18	0,91	46852	33756	28,0
19	0,89	26715	34049	27,5
20	0,81	25132	35641	41,8
21	0,76	53544	36844	31,2
22	0,73	45672	37542	17,8
23	0,69	61605	38709	37,2
24	0,70	61490	38348	37,6
25	0,67	74472	39360	47,1
26	0,65	47206	39852	15,6
27	0,65	56496	40037	29,1
28	0,63	39849	40389	1,4
29	0,58	28565	42149	47,6
30	0,60	50445	41396	17,9
31	0,58	50306	42376	15,8
32	0,53	50468	44082	12,7
33	0,47	58323	46985	19,4
34	0,44	53009	48349	8,8
35	0,42	55370	49369	10,8

36	0,46	58114	47258	18,7
37	0,43	65951	48909	25,8
38	0,45	39508	47754	20,9
39	0,39	80315	51529	35,8
40	0,41	53429	50261	5,9
41	0,41	50468	50522	0,1
42	0,34	52125	55346	6,2
43	0,34	47300	55506	17,3
44	0,34	46925	55063	17,3
45	0,33	68811	56013	18,6
46	0,32	76774	56650	26,2
47	0,29	55686	59994	7,7
48	0,27	46368	61882	33,5
49	0,28	24517	61320	150,1
50	0,28	58332	60432	3,6
51	0,28	62698	60997	2,7
52	0,22	50182	68567	36,6
53	0,21	61555	70319	14,2
54	0,24	84931	65429	23,0
55	0,19	27170	74355	173,7
56	0,21	80281	70423	12,3
57	0,17	76787	77525	1,0
58	0,16	85419	79911	6,4
59	0,15	55612	83896	50,9
60	0,13	66369	90245	36,0
61	0,10	39021	102068	161,6
62	0,07	46723	124751	167,0
	Means	49217	46827	33,8

Lesson 4: Discovery and use of the *Wave nature of Light WNL*

Goals: The students can illustrate the WNL and use it for the identification of discrete spectral lines, especially the H_α-line.

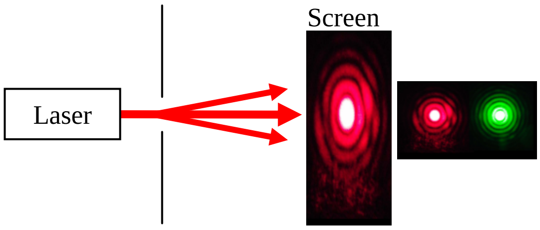
Time	Didactical Com-ments	Methodical Com-ments	Social form
5	Introduction: Question, see below	Teacher presents the question	Discussion
10	Ideas: see below	Students suggest	,,
15	Instruction: Dif-fraction experi-ment	Students describe the diffraction pattern and sug-gest the wave nature of light	,,
40	Instruction: Meas-urement of the wavelengths of a hydrogen lamp and of a neon lamp	Students calculate the wavelengths according to a given procedure	Small Groups
60	Development of results: Determin-ing the matter on Vega	Determination of the spectrum with Spectrarium, identification of Hydrogen by comparison	Small Groups
75	Formulation of results: see below	Formulation and animation in the plenum	Student-presentation
90	Consolidation: observation of spectra with the hand-spectrometer	For experts: calculation	Small Groups

Intended Blackboard Figure

How Can We Measure the Dominant Matter on Vega?

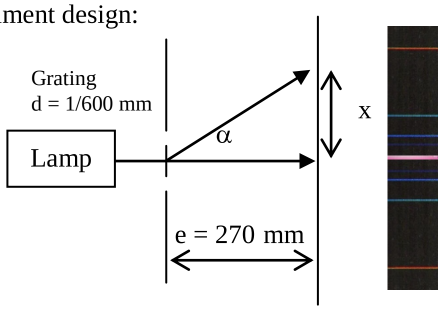
Ideas: no probes → analysis of light

An experiment illustrates: Light propagates like a wave and has a wavelength λ. $\lambda_{red} > \lambda_{green}$



Measurement of the Wavelength with a Grating

Experiment design:



colour	violet	blue	cyan	red
x in mm	68	73	82	116

Results: The dominant matter on Vega is hydrogen. We can identify this by the discrete hydrogen lines, especially by the red H_α-line with λ=656 nm.

Calculation for experts:

Red: $\lambda = d \cdot \sin[\arctan(x/e)] = 658$ nm (Literature: 656.3 nm)

Cyan: $\lambda = d \cdot \sin[\arctan(x/e)] = 484$ nm (Literature: 486.1 nm)

Blue: $\lambda = d \cdot \sin[\arctan(x/e)] = 435$ nm (Literature: 434.1 nm)

Violet: $\lambda = d \cdot \sin[\arctan(x/e)] = 407$ nm (Literature: 410.2 nm)

Lesson 5: Discovery and use of the redshift

Goals: The students can explain the redshift and use it for the determination of velocities.

Time	Didactical Com-ments	Methodical Com-ments	Form
8	Introduction: NGC3516 in Stel-larium	Students suggest measurements of D and spectrum	Discus-sion
10	Development of the question	Question of the lesson	Discus-sion
15	Hypotheses: see below	Conjecture	,,
20	Animation: see below	Students determine λ with Spectrarium	Small Groups
22	Development of the question	Question of the lesson	Discus-sion
25	Hypotheses: see below	Conjecture	,,
40	Development of results: performing the control experi-ment	Students perform experiment	Small Groups
45	Formulation of	Formulation and	Student-

	results: see below	animation in the plenum	presentation
55	Development of results: instruction of v=c·z	Students calculate v	Instruction, Calculation
90	Consolidation: Derivation, investigation of other galaxies	Students work with worksheet	Small Groups

Intended Blackboard Figure

At what Wavelength is the H_α – Line in NGC3516?

Hypotheses: 656 nm

Spectrarium: 662 nm Surprise!

Why is the H_α – line shifted in NGC3516?

Hypothesis: Motion changes the wavelength

Control experiment:

- We move a toy duck in water
- Behind the duck the wavelength is increased
- In front of the duck the wavelength is decreased
- For the case of light an increase of λ means a shift towards red and is called redshift.

Results:

- The H_α – line of NGC3516 shows a redshift. This is a consequence of an increase of the distance with time.
- Also the distance to other distant galaxies increases with time.

Calculation of the velocity: v = c·z; z = Δλ/λ = 0,009 → v = 2700 km/s

Intended solution of the worksheet:

1) The wavelength increases behind the duck.
   For experts:
2) λ = 5 cm → c = 1 cm/s
3) λ‘ = 7 cm; s = 2 cm → λ‘ – λ = 2 cm = s
4) v = s/T = Δλ/T = Δλ/λ · λ/T = z·c
5) We take a spectrum, identify the redshift z, multiply it by c and obtain v.

Worksheet, Astronomy Club, Dr. Carmesin 2013

Wavelength of a Moving Source

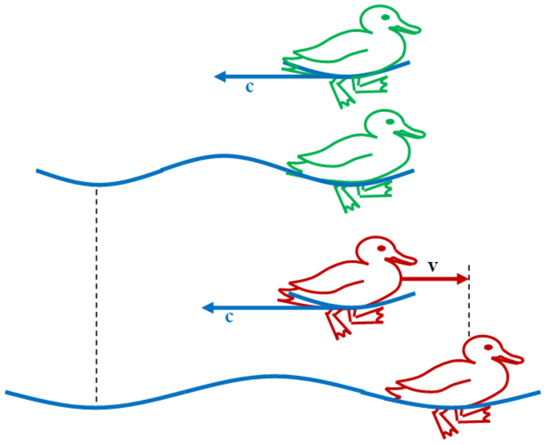
Exercise 1: Describe the wavelengths of the swimming baby duck.

The green duck cannot move forward, though it moves its legs. The wave moves with its own velocity c. After a period T = 5s, the wave-minimum has moved backwards one wavelength λ.

Exercise 2: Measure λ and calculate the velocity c of the wave.

The red duck moves forward with its velocity v. After a period T = 5s the wave-minimum has moved backwards one wavelength λ. Meanwhile the duck has moved forward the distance s.

Exercise 3: Measure the new wavelength λ‘ as well as the distance s and find a relation between λ‘ , λ and s.



Exercise 4: Express the velocity v by c and the redshift z = Δλ/λ.

Exercise 5: Plan a procedure with which you can measure the velocity v of the moving source with help of the redshift z and the propagation velocity c.

Lesson 6: Discovery of the Equality of Start Times EST and Measurement of the Age of the Universe

Goals: The students can explain their discovery of the EST and measure the age τ of the universe.

Time	Didactical Comments	Methodical Comments	Social form
8	Introduction: D, v, see below	Pupils explain	Discussion
10	Development of the question	Main question of the lesson	Discussion
20	Plans: see below	Plan	„
30	Development of result: τ	Calculation	Small Groups
35	Formulation of result: see below	Formulation	Pupil-presentation
40	Development of the 2. question	2. question	Discussion
60	Development of result: τ	Calculation	Small Groups
65	Formulation of result: see below	Formulation	Pupil-presentation
80	Metacognition: see below	Interpretation, Reflexion	Discussion
90	Consolidation: Investigate the proportionality	Worksheet	Small Groups

Intended Blackboard Figure

NGC3516: D = 1300 Zm; v = 2700 km/s

When Did NGC3516 Start?

Plans: We determine D and v and calculate t = D/v.

D in Zm; v in Zm/Gy

Common units: v = 2700 km/s = 2.7 Mm/s

| 1y = 31.6 Ms; v = 85 Tm/y = 85 Zm/Gy

Starting time: τ = 1300 Zm/(85 Zm/Gy) = 15 Gy

Result: The distance was zero 15 Gy ago. The starting time of NGC3516 is 15 Gy.

	D in Zm	v in Zm/Gy	τ in Gy
NGC3516	1300	85	15

NGC2276	1200	75	16
M66	400	23	17
M74	440	28	16
M83	210	16	13
M84	460	32	14
M105	460	29	16
Mean			15.3

Result: All galaxies have the same starting time within the accuracy of the observation.
The common starting time is 15.3 Gy (Literature 13.8 Gy).
Interpretation: Everything began to expand 13.8 Gy ago. This is called the Big Bang. The age of the universe is 13.8 Gy.

Worksheet, Astronomy Club, Dr. Carmesin 2013

NGC2276: HST 22.9.2009.
M66: VLT 7.1.2009 by ESO

The following apparent brightnesses and redshifts can be simulated:

Galaxy	m	z
NGC3516	11.6	0.0087
NGC2276	11.4	0.0079
M66	9.0	0.0024
M74	9.2	0.0029
M83	7.6	0.0017
M84	9.3	0.0034
M105	9.3	0.003

Exercise: Calculate the starting times τ.

5. Mathematical Level

So far the students investigated the age of the universe using the concept that distant galaxies move away from the earth and that the velocity v is proportional to the distance d thereby. Further investigation shows however that geometry and gravity should be considered in more detail. This has been treated in a third unit with students from classes 10 to 12 as follows:

Lesson 1: Exclusion of a possible centre of the universe

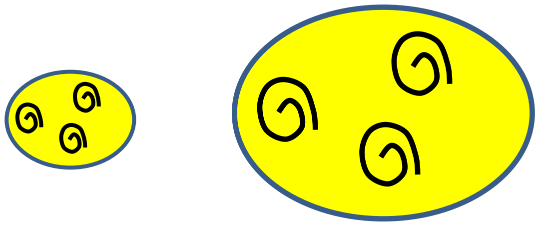
Goals: The students can explain that the motion of distant galaxies looks similar from each point of the universe.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Motion of distant galaxies	Students describe	Discussion
10	Development of the question	Question of the lesson	Discussion
15	Hypotheses: see below	Conjecture	,,
20	Experiment: Plan see below	Planning	,,
30	Performing the experiment	Students blow up the balloon	Small Groups
45	Formulation of results: see below	Formulation and demonstration in the plenum	Pupil-Presentation

Intended Blackboard Figure

Is the earth at the centre of the universe?

Hypotheses: yes, because distant galaxies move away from the earth
No, because it is unlikely.
Model experiment: Balloon with galaxies drawn on it. Balloon is blown up.



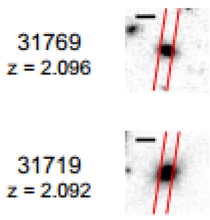
Results: Each inhabitant of a galaxy observes, that the other galaxies move away from him. So the earth is not the centre of the world.

Lesson 2: Discovery of the expansion of space

Goals: The students can argue that the space expands.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Figure	Students describe	Discussion
10	Development of the question	Question of the lesson	Discussion
15	Hypotheses: see below	Conjecture	,,
20	Experiment: Plan see below	Planning	,,
30	Performing the experiment	Students blow up the balloon	Small Groups
45	Formulation of results: see below	Formulation and demonstration	Pupil-Presentation

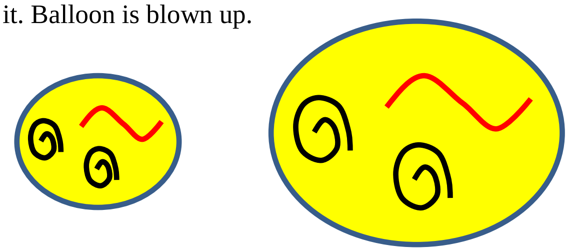
Introductory Figure:



Intended Blackboard Figure [4]

Do galaxies move faster than light?

Hypotheses: yes, because $v = z \cdot c$
No, because nothing moves faster than light
Model experiment: Balloon with galaxies drawn on it. Balloon is blown up.



Correspondence

Model Experiment	Universe
Spiral	galaxy
Expanding balloon	expanding space
Spiral resting on balloon	galaxy resting in space
Waveline expands	wavelength expands

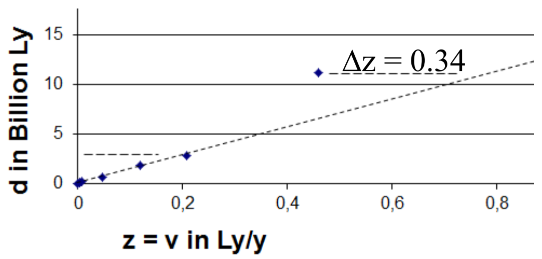
Results: The measured redshift z indicates the expansion of space. As a result the distance d between galaxies grows. Thereby the increase of distance per time $\Delta d/\Delta t$ may become larger than c.

Lesson 3: Discovery of the accelerated expansion

Goals: The students can argue that the expansion is accelerated.

Time	Didactical Comments	Methodical Comments	Form
10	Introduction: Linear graph for d(z)	Students draw and present	Small Groups
15	Development of the question	Question of the lesson	Discussion
25	Ideas	Students suggest	Discussion
35	Consequences	Students argue	Small Groups
45	Formulation of results: see below	Formulation and argumentation	Pupil-Presentation

Introductory Figure:



Intended Blackboard Figure [7,8]

Linear graph for d(z)

How do very distant galaxies move?

Ideas:

Light from very distant galaxies was emitted long ago.

Small redshift corresponds to small velocity.

Results:

The expansion is accelerated. Since the velocity was too small in earlier times, if compared to a constant velocity.

Lesson 4: Exclusion of acceleration by the gravity of masses

Goals: The students can exclude that the expansion is accelerated by the gravity of the masses.

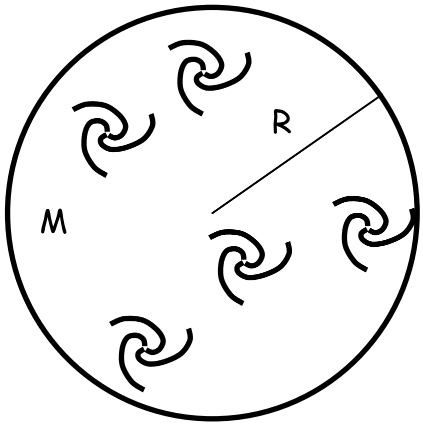
Time	Didactical Comments	Methodical Comments	Form
5	Introduction: accelerated expansion	Students describe it	Discussion
10	Development of the question	Question of the lesson	Discussion
15	Hypotheses	Students conjecture	Discussion
25	Model and Situation	Teacher explains the model	Instruction
80	Energies	Calculation, conclusions	Small Groups
90	Formulation of results: see below	Formulation and argumentation	Pupil-Presentation

Intended Blackboard Figure

Why is the motion accelerated?

Hypotheses: Gravity of galaxies

Electric force: galaxies are not charged



Model: galaxies with mass M are in a sphere with radius R. The sphere expands. Relevant are the kinetic and gravitational energies.

$E = - m\cdot M\cdot G/R + m\cdot v^2/2$

Results:

The energy is constant. While the distances increase, the radius R increases and the velocities v decrease. So the usual gravity of masses does not explain the observed accelerated expansion.

Lesson 5: Discovery of two typical motions

Goals: The students can explain the two typical motions due to the gravitation of masses.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Model	Students explain	Discussion
10	Development of the question	Question of the lesson	Discussion
25	Kickers	Teacher explains	Instruction
70	Energies	Calculation, comparison	Small Groups
90	Formulation of results: see below	Formulation and demonstration	Pupil-Presentation

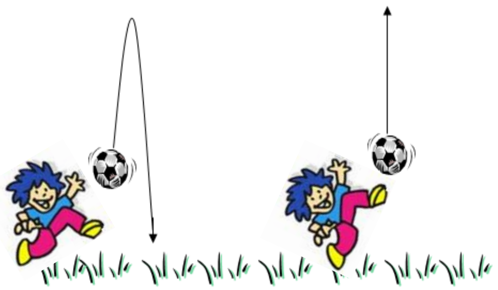
Intended Blackboard Figure

How does the radius R develop as a consequence of the gravity of the galaxies?

Ideas: $E = - m\cdot M\cdot G/R + m\cdot v^2/2$

Slowing down

Kickers: Students kick a ball vertically upwards.



$E = - m \cdot M \cdot G/R + m \cdot v^2/2$
Correspondence

Kickers	Model of universe
Ball with mass m	galaxy with mass m
Mass M of earth	mass M of galaxies in sphere
Increasing height	increasing distances
Velocity v	$v = \Delta R/\Delta t$

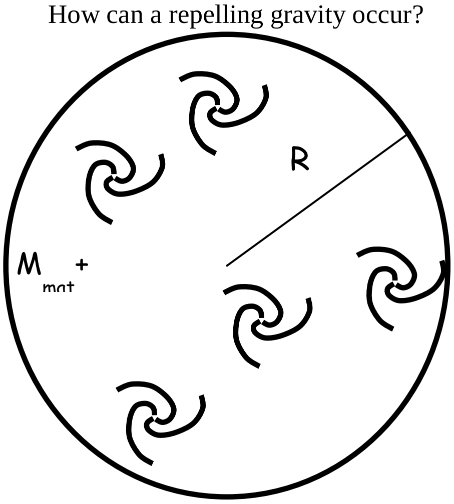
Results:
The energy terms are equal, so the motions are equal. The ball may exhibit one of two typical motions:
$E < 0 \rightarrow$ ball comes back
$E > 0 \rightarrow$ ball never comes back
So the universe may exhibit two typical motions:
$E < 0 \rightarrow$ universe expands and then contracts
$E > 0 \rightarrow$ universe expands for ever

Lesson 6: Discovery of repelling gravity

Goals: The students can explain how a density of the vacuum causes a repelling gravity.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Model	Students explain	Discussion
10	Development of the question	Question of the lesson	Discussion
20	Constant density of the vacuum	Teacher suggests	Instruction
70	Energies	Calculation, graph, conclusion	Small Groups
90	Formulation of results: see below	Formulation and demonstration	Pupil-Presentation

Intended Blackboard Figure
How can a repelling gravity occur?



Ideas: E(R) = ?
$M_V = \rho_V \cdot V = \rho_V \cdot 4\pi/3 \cdot R^3$
$E/m = - M_{Mat} \cdot G/R - G \cdot \rho_V \cdot 4\pi/3 \cdot R^3/R + v^2/2$
$E/m = - M_{Mat} \cdot G/R - G \cdot \rho_V \cdot 4\pi/3 \cdot R^2 + v^2/2$
Results:
For small R, the -1/R-term dominates and the gravity acts attractive.
For large R, the $-R^2$-term dominates and the gravity acts repulsive.

Lesson 7: Calculation of critical density

Goals: The students can calculate the critical density for the acceleration.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Model	Students explain	Discussion
10	Development of the question	Question of the lesson	Discussion
15	Ideas	Students develop	Discussion
40	E'/m	Calculation, comparison	Small Groups
45	Formulation of results: see below	Formulation and Derivation	Pupil-Presentation

Intended Blackboard Figure
At what density does the acceleration occur?
Ideas: E'=0; F = -E'
Results:
$F/m = - M_{Mat} \cdot G/R^2 + G \cdot \rho_V \cdot 8\pi/3 \cdot R = R''$ |:R
$R''/R = G \cdot 4\pi/3 \cdot (2\rho_V - \rho_M) \rightarrow$ v increases, if $2\rho_V > \rho_M$
Notation: Friedmann-Lemaitre-Equation 1

Lesson 8: Interpretation of the energy

Goals: The students can interpret the energy by curvature of space.

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Model with energy term E	Students explain	Discussion
10	Development of the question	Question of the lesson	Discussion
20	Information	Teacher informs	Instruction
40	Analogy	Students suggest, teacher helps	Discussion
70	Derivation	Students derive	Small Groups
90	Formulation of results: see below	Formulation and Derivation	Pupil-Presentation

Intended Blackboard Figure
How can we interpret the energy?
Information:
Newtonian Dynamics: path curved by gravity
Einstein-Dynamics: straightest path for $v \rightarrow c$, space curved by gravity
Analogy:

E	**path of comet**	**space**
$E < 0$	elliptical, closed	closed & curved
$E > 0$	hyperbolic, open	open & curved
$E = 0$	littler curvature	little curvature

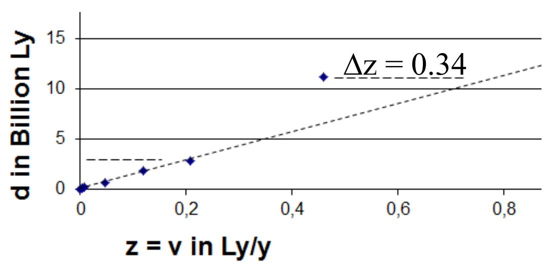
Derivation:
Goal: Term fpr $(R'/R)^2$
Abbreviation: $k=-2E/(m\cdot c^2)$
$E/m = -M_{Mat}\cdot G/R - G\cdot\rho_V\cdot 4\pi/3\cdot R^2 + R'^2/2 \quad |\cdot 2/R^2$
$(R'/R)^2 = G\cdot 8\pi/3\cdot(\rho_V+\rho_M)+2E/(mR^2)$
$(R'/R)^2 = G\cdot 8\pi/3\cdot(\rho_V+\rho_M)-kc^2/R^2$
Notation: Friedmann-Lemaitre-Equation 2

Lesson 9: Evaluation of observations

Goals: The students can determine the densities of matter and vacuum from observations

Time	Didactical Comments	Methodical Comments	Form
5	Introduction: Model and observation	Students explain	Discussion
10	Development of the question	Question of the lesson	Discussion
30	Scaling	Teacher informs	Instruction
70	Calculation	Students calculate according to worksheet	Small Groups
90	Formulation of results: see below	Formulation and Derivation	Pupil-Presentation

Introductory Figure:



Intended Blackboard Figure
How large are the densities of matter and vacuum?
Ideas: $(\rho_M|\rho_V|k)=?$ Three equations from data
Scaling: $\Omega = \rho/\rho_s$ with $\rho_s\cdot G\cdot 4\pi/3 = 0.0028/Gy^2$
$(\rho_s=10^{-26}kg/m^3)$

$(R'/R)^2=0.0056/Gy^2\cdot(\Omega_V+\Omega_M)-kc^2/R^2$
$R''/R=0.0028/Gy^2\cdot(2\Omega_V-\Omega_M)$
Equations:
$R''/R = (\Delta v/\Delta t)/R = (0.34c/11.15Gy)/11.15GLy = 0.0027/Gy^2$
→ $0.96= 2\Omega_V - \Omega_M$ (1)
$R'/R = 0.46c/11{,}15GLy + 11.15\ Gy\cdot 0.0027/Gy^2 = 0.071/Gy$
→ $0.89=\Omega_V+\Omega_M-1.4k$ (2)
$R'/R = 0.2c/2.8GLy + 2.8\ Gy\cdot 0.0027/Gy^2 = 0.079/Gy$
→ $1.1=\Omega_V+\Omega_M-23k$ (3)
Solutions:
$(\Omega_M|\Omega_V|k) = (0.26|0.61|-0.009)$
$\rho_M=0.26/(0.61+0.26)=30\%$
$\rho_V=70\%$
$k\approx 0$
$\rho_{dark\ matter}=25\%$
$\rho_{visible}=5\%$

Worksheet, Astronomy Club, Dr. Carmesin 2012
Exercise 1: Calculate from the upper data point $(d|z|\Delta z) = (11.15\ GLy|0.8|0.34c)$ a term for $R''/R = (\Delta v/\Delta t)/R = \ldots$ and insert into the Friedmann-Lemaitre-Equation 1.
Exercise 2: Calculate from the upper data point $(d|z|\Delta z) = (11.15\ GLy|0.8|0.34c)$ a term for $R'/R = v/R + t\cdot v'/R = v/R + t\cdot R''/R = \ldots$ and insert into the Friedmann-Lemaitre-Equation 2.
Exercise 3: Calculate from the lower data point $(d|z|\Delta z) = (2.8\ GLy|0.2|0)$ a term for $R'/R = v/R + t\cdot v'/R = v/R + t\cdot R''/R = \ldots$ and insert into the Friedmann-Lemaitre-Equation 2.
Exercise 4: Calculate $(\Omega_M|\Omega_V|k)$ and $(\rho_M|\rho_V|k)$.

6. Experiences

The students enjoy very much to work and play with the various applets by their own. At the conceptual level pupils starting from classes five dealt well with the simple numbers, the evaluation of the age of the universe and the explanation of the equal starting times. The disadvantage of this easy learning process is that the students could hardly imagine how realistic measurements look like. At the expert level pupils from class six made model experiments and simulated measurements in a concentrated manner. The learning process became more difficult, but the pupils enjoyed participating in the measuring process. The disadvantage of the full concentration in the measuring process was that the pupils imagined everything happening in a fixed three dimensional space rather than in an expanding space. At the mathematical level the students from class 10 used their competence in argumentation and gravity. Some needed a repetition in Newtonian gravitation theory first. On this basis the students enjoyed to draw conclusions based on data, derivations or argumentation by themselves. They presented their results and explanations on a public astronomy evening and by giving a talk at the MNU-Bundeskongress in Hamburg in 2013. At the teacher training in Lisbon in 2013, the teachers enjoyed to make their own experiments and to discuss open questions at a high level. After the training was evaluated it turned out that most teachers liked the presentations, the concept, the experiments, the lessons as well as the worksheets.

7. Development of Personality

The pupils experience norms in the family, school, city, local community and state they live in. Accordingly they develop their personality [9]. However the pupils understand the partial arbitrariness of these norms and need a wider range of experiences. Such a wider range is offered by astronomy and especially by cosmology - however books and stories do not provide the necessary experiences. With the presented teaching units the pupils can explore model experiments as well as applets about cosmology and make their own reliable experiences in a wide intellectual range. So the pupils can make a further step in their personal development.

8. Conclusion

The students learnt with hands on experiments including applets in the presented lessons. So we achieved high learning efficiency, as expected [1]. The lessons can easily be prepared, because the outlines and worksheets are presented here, the model experiments are simple and the applets are available as freeware.

9. Literature

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