Students Learn to Solve the Cosmological Constant Problem

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Abstract

In everyday life, digital cameras and photovoltaic systems are ubiquitous. Thereby, quanta of the electromagnetic field are absorbed in order to gain visual information or electric energy. Our understanding of such quanta is rooted on the quantum field theory of electrodynamics, quantum electrodynamics, QED, in present-day physics. However, that QED predicts an energy density of the vacuum of $u_{\Lambda,\mathrm{QED}} = 3.6 \cdot 10^{112}\, J/m^3$. In general, the energy density of space u_Λ is related to the cosmological constant Λ proposed by Einstein (1917). In contrast to $u_{\Lambda,\mathrm{QED}}$, the energy density of space, the dark energy, has been observed at the intergalactic space: $u_{\Lambda,\mathrm{obs}} = 5.1 \cdot 10^{-10}\, J/m^3$. That huge discrepancy presents a severe problem of QED, it is called the cosmological constant problem. How is that problem resolved?

This question is answered with help of the dynamics of volume in nature, the volume dynamics, VD, see Carmesin (2023a). The VD bridge general relativity and quantum physics. For these results, we provide a learning process, so that you can directly use the concept in your courses. The learning process has been tested in various learning groups, and experiences are reported.

1. Introduction

In order to understand the meaning of the energy density associated with space or volume, we analyse Einstein's (1917) idea of a cosmological constant Λ.

1.1. On Einstein's idea of Λ

The expansion of space can be derived from general relativity, see Einstein (1917), Friedmann (1922) and Lemaître (1927). Thereby, a uniform scaling of space is derived. In general, such a uniform scaling can be described by the time evolution of a scale radius $r(t)$, see Fig. (1): If space expands by a factor q, then r is multiplied by q. That time evolution can be described by this differential equation, DEQ:

$$\frac{\dot{r}^2}{r^2} = \frac{8\pi G}{3} \cdot (\rho_r + \rho_m + \rho_K + \rho_\Lambda) \qquad \{1\}$$

Hereby, G is the universal constant of gravity. Moreover, four densities are distinguished, so that each density has a characteristic scaling behaviour as a function of the scale radius r:

ρ_r is the density of radiation,

ρ_m is the density of matter,

ρ_K is the density of a curvature parameter, it is zero according to observation, see Planck collaboration (2020), and as a result of a proof, see Carmesin (2023c),

ρ_Λ is the density of the cosmological constant, it does not change as a function of the scale radius r.

A present-day value of a quantity is marked by the subscript zero. Next, the densities in Eq. {1} are expressed as functions of the scale radius:

$$\frac{\dot{r}^2}{r^2} = \frac{8\pi G}{3} \cdot \left(\rho_{r,0}\frac{r_0^4}{r^4} + \rho_{m,0}\frac{r_0^3}{r^3} + \rho_\Lambda\right) \qquad \{2\}$$

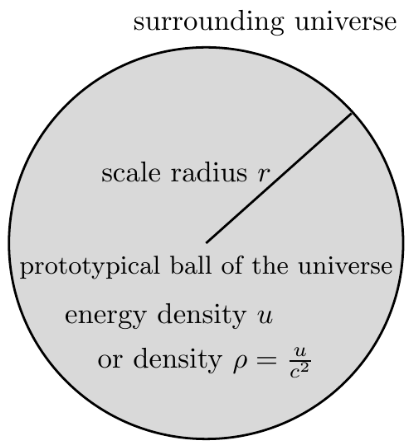

Fig. 1: A prototypical ball of the universe with a scale radius r and an energy density u. The energy density can be expressed in terms of a density or dynamic density $\rho = \frac{u}{c^2}$.

When ρ_Λ becomes essential, r is very large, so that ρ_r becomes very small, so we neglect it in section (1.1). We multiply by r^2 and apply the time derivative:

$$\frac{\partial}{\partial t}\dot{r}^2 = \frac{8\pi G}{3} \cdot \frac{\partial}{\partial t}\left(\rho_{m,0}\frac{r_0^3}{r^1} + \rho_\Lambda r^2\right) \qquad \{3\}$$

$$2\dot{r}\,\ddot{r} = \frac{8\pi G}{3}\left(-\rho_{m,0}\frac{r_0^3}{r^2} + 2\rho_\Lambda r\right)\dot{r} \qquad \{4\}$$

In order to obtain a relative acceleration $\frac{\ddot{r}}{r}$, we divide by $2r\dot{r}$:

$$\frac{\ddot{r}}{r} = \frac{8\pi G}{3}\left(-\frac{1}{2}\rho_m + \rho_\Lambda\right) \qquad \{5\}$$

Einstein (1917) had the idea of a static universe: If the ρ_Λ compensates $\frac{1}{2}\rho_m$ in the above DEQ, then r is not accelerated. Thus, if $\dot{r}$ is zero initially, then $\dot{r}$ remains zero and the universe is static.

For this purpose of a possibly static universe, Einstein (1917) proposed the cosmological constant Λ, corresponding to the density $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$ and energy density $u_\Lambda = \frac{\Lambda c^4}{8\pi G}$, see e. g. Hobson (2006, section 15.1).

1.2. **Epistemology**

Kircher, Girwidz und Häußler (2001, section 4.1.2) describe the hypothetic deductive method. In the epistemological literature, this method is also called hypothetico-deductive testing (Niiniluoto, Sintonen, Wolenski 2004, S. 214). The method consists of three steps: In the hypothetic step, a thesis or hypothesis is suggested for testing. In the deductive step, implications are derived. In the third step, the implications are compared with observation. Hereby, in principle, a falsification should be possible. This method is used here as well as in Carmesin (2024a-g, 2017, 2018a-b, 2019a-b, 2020a-c, 2021a-d, 2022a-c, 2023a-f).

1.3. **On the observed value Λ_{obs}**

As a consequence of Eq. {5}, it was clear how ρ_Λ could be measured: If an observer would measure an accelerated expansion of space, then this could be explained by the dynamic density ρ_Λ, see e. g. Carmesin (2019a, 2020a). Indeed, Perlmutter et al. (1998) discovered the accelerated expansion of the universe.

Meanwhile, many observers confirmed the accelerated expansion of the universe. An especially precise measurement of ρ_Λ has been achieved with help of the cosmic microwave background, CMB, see Planck collaboration (2020). That group applied several evaluation procedures, whereby the so-called temperature-temperature correlation is especially robust and used here:

The Hubble constant H_0 is the present-day value of the Hubble parameter $H = \frac{\dot{r}}{r}$, the observed value is:

$$H_{0,obs} = 66.88\ (\pm 0.92)\ \frac{\mathrm{km}}{\mathrm{s\cdot Mpc}} \quad \text{with}$$

$$1Mpc = 3.086 \cdot 10^{19}\ \mathrm{km}, \qquad \text{thus,}$$

$$H_{0,obs} = 2.167\ (\pm 0.03) \cdot 10^{-18}\ \frac{1}{\mathrm{s}} \qquad \{6\}$$

With it, the so-called critical density is as follows:

$$\rho_{cr.} = \frac{3H_0^2}{8\pi G} = 8.4 \cdot 10^{-27}\ \frac{\mathrm{kg}}{\mathrm{m^3}} \qquad \{7\}$$

The density divided by the critical density is the density parameter, $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$. Its observed value is:

$$\Omega_{\Lambda,obs} = 0.679\ (\pm 0.013) \qquad \{8\}$$

Note that this density parameter means that 67.9 % of all energy and matter in the universe is the energy of ρ_Λ, the so-called dark energy, see Huterer (1999), Planck collaboration (2020), Workman et al. (2022).

Thus, the observed value of ρ_Λ is:

$$\rho_{\Lambda,obs} = \Omega_{\Lambda,obs} \cdot \rho_{cr.} = 5.704\ (\pm 0.27) \cdot 10^{-27}\ \frac{\mathrm{kg}}{\mathrm{m^3}} \qquad \{9\}$$

$$u_{\Lambda,obs} = 5.133\ (\pm 0.243) \cdot 10^{-10}\ \frac{\mathrm{J}}{\mathrm{m^3}} \qquad \{10\}$$

1.4. **On photon states**

How are the photons described that the camera chip of a digital camera absorbs: In QED, photons with a circular frequency ω_μ are described by orthonormal photon states $|n_{\mu,p}\rangle$, with $n \in \{0, 1, 2, 3, ...\}$:

$$\langle n_{\mu,p}|n_{\mu',p}\rangle = \delta_{\mu,\mu'} \qquad \{11\}$$

Hereby, the subscript p marks photons. These states $|n_{\mu,p}\rangle$ are eigenfunctions of the number operator $\widehat{N}_{\mu,p}$:

$$\widehat{N}_{\mu,p}|n_{\mu,p}\rangle = n_{\mu,p}|n_{\mu,p}\rangle \qquad \{12\}$$

The energy operator is as follows:

$$\widehat{H} = \sum_\mu \left(\widehat{N}_{\mu,p} + \frac{1}{2}\right) \cdot \hbar\omega_\mu \qquad \{13\}$$

The matrix elements of $\widehat{H}$ are as follows:

$$H_{\mu,\mu'} = \left|n_{\mu,p}\right\rangle \left(n_\mu + \frac{1}{2}\right) \hbar\omega_\mu \delta_{\mu,\mu'} \delta_{p,p'} \langle n_{\mu',p}| \quad \{14\}$$

The Kronecker delta $\delta_{\mu,\mu'}$ indicates that only the diagonal matrix elements are nonzero. The Kronecker delta $\delta_{p,p'}$ indicates that quantum states $\langle n_{\mu',p'}|$ with $p \neq p'$ of other quanta than photons provide a factor zero.

Consequently, zero photon state $|0_{\mu,p}\rangle$ of the circular frequency ω_μ has the following eigenvalue equation for the energy:

$$\widehat{H}\left|0_{\mu,p}\right\rangle = \left(\widehat{N}_{\mu,p} + \frac{1}{2}\right) \hbar\omega_\mu \left|0_{\mu,p}\right\rangle = \frac{1}{2}\hbar\omega_\mu |0_{\mu,p}\rangle \quad \{15\}$$

Thus, the eigenvalue of the energy is nonzero. It is called the zero-point energy:

$$ZPE_{\mu,p} = \frac{1}{2}\hbar\omega_\mu \qquad \{16\}$$

Correspondingly, the zero photon state $|0_{\mu,p}\rangle$ is the zero-point oscillation, ZPO, of the electromagnetic field at the circular frequency ω_μ.

If there is a state $|n_{\mu,p}\rangle$, and if an additional photon is emitted, for instance by an LED, then the number state is increased by one, so that the state $|n_{\mu,p}+1\rangle$ occurs. This process is described with help of a raising operator $\hat{a}^{+}_{\mu,p}$ as follows:

$$\frac{1}{\sqrt{n_{\mu,p}+1}}\cdot\ \hat{a}^{+}_{\mu,p}\left|n_{\mu,p}\right\rangle = |n_{\mu,p}+1\rangle \qquad \{17\}$$

The process of emission is expressed in the form of an reaction equation as follows:

$$\hbar\omega_{\mu} + \left|n_{\mu,p}\right\rangle = |n_{\mu,p}+1\rangle \qquad \{18\}$$

Hereby, the energy $\hbar\omega_{\mu}$ must be provided by the emitting device in the form of a photon. Similarly, if the camera chip absorbs a photon at a state $|n_{\mu,p}\rangle$, the prosses can be described by the lowering operator $a_{\mu p}$ as follows:

$$\frac{1}{\sqrt{n_{\mu,p}}}\cdot\hat{a}_{\mu,p}\left|n_{\mu,p}\right\rangle = |n_{\mu,p}-1\rangle \qquad \{19\}$$

The process of absorption is expressed in the form of an reaction equation as follows:

$$\left|n_{\mu,p}\right\rangle = \left|n_{\mu,p}-1\right\rangle + \hbar\omega_{\mu} \qquad \{20\}$$

Hereby, the energy $\hbar\omega_{\mu}$ of one photon must be taken up by the absorbing device.

In this manner, an LED can increase the number of photons in the state $= |n_{\mu,p}\rangle$, whereby the circular frequency ω_{μ} corresponds to the colour of the LED. Analogously, a colour – pixel corresponding to ω_{μ} can decrease the number of photons in the state $|n_{\mu,p}\rangle$.

So far, QED describes the emission and absorption of photons in a very intuitive manner that is also in precise accordance with observation, see e. g. Ballentine (1998), see also Carmesin (2021a, 2023a, 2024a-d) for a derivation of the above algebra.

However, the zero-point energy corresponds to an energy density of the vacuum of $u_{\Lambda,\mathrm{QED}} = 5.9\cdot 10^{111}\,J/m^3$, see e. g. Ballentine (1998). This is in clear contrast to the energy density of intergalactic space of $u_{\Lambda,\mathrm{obs}} = 5.1\cdot 10^{-10}\,J/m^3$. This huge discrepancy is called cosmological constant problem, CCP.

1.5. **Aim of the paper**

The aim of the paper is to show how students or interested people can derive a solution to the CCP.

1.6. **Organization of the paper**

A didactic analysis including a professional analysis is provided in section 2. The learning process including experiences with learning groups are shown in part 3. We discuss our findings in section 4. Many useful and insightful related results are presented in my parallel papers in the report about the DPG conference in March 2024 in Greifswald, see Carmesin (2024a-g).

2. **Didactic analysis**

In a first didactic step in section (1.1), Einstein's (1907) introduction of the cosmological constant is presented. Thereby, the expansion of space according to Eq. {1} has been treated before. On that basis, this step has no special learning barrier. This step is essential, in order to have a clear concept of the cosmological constant and its density.

In a second didactic step in section (1.3), the observed values are presented. This step has no special learning barrier. The step is essential in order to understand the cosmological constant problem, CCP.

In a third didactic step in section (1.4), the ladder operators and number states are introduced for the case of electromagnetic radiation. This step is intuitive. Spectra are well-known from atoms, for instance, see e. g. Carmesin (2020c). It is similar to the states Thus, this step has no special learning barrier. The step is essential in order to explain why the volume does contribute to ρ_{Λ}, but the electromagnetic radiation does not.

2.1. **Derivation of the observed $\mathbf{u}_{\Lambda,obs}$**

2.1.1. **Physical analysis**

Carmesin (2023a, 2024a-f) analysed the dynamics of the volume in nature, the volume-dynamics, VD. It includes the local formation of volume, LFV. With it, Carmesin (2021a, 2023a, 2024c) derived the energy density of volume as follows:

Theorem: Law of the derived energy density of volume in an empty universe.

In a universe consisting of volume only, the process of GFV from LFV causes the following energy density of volume:

$$u_{\Lambda,\mathrm{theo}} = \frac{c^2H_0^2}{4\pi G} = u_{\mathrm{vol}}, \quad \text{thus,} \qquad \{21\}$$

$$\rho_{\Lambda,\mathrm{theo}} = \frac{H_0^2}{4\pi G} = \ 5.600\ (\pm 0.155)\cdot 10^{-27}\,\frac{\mathrm{kg}}{\mathrm{m}^3} \quad \{22\}$$

The density is a consequence of the process of formation of volume since the Big Bang until the present-day time t_0.

If that process ranges from the Big Bang to another time $t_1 \neq t_0$, then that process provides the same density of volume.

This result is in precise accordance with observation.

Proofs are presented in Carmesin (2021a, 2023a, 2024c-d). This result provides a solution to the CCP.

Next, we analyse, why the energy density $u_{\Lambda,\mathrm{QED}}$ does not contribute to the energy density $\mathrm{u}_{\Lambda,obs}$ observed at the intergalactic space, see Perlmutter (1998).

2.1.2. **Didactic analysis**

In a first didactic step, the derived energy density of volume is presented and discussed. That derivation and the corresponding didactic steps are analysed in Carmesin (2024c).

2.2. $u_{\Lambda,\text{QED}}$ does not contribute to the density ρ_r

2.2.1. Physical analysis

The density ρ_r in Eq. {1} represents the classical density of the electromagnetic radiation and of possible other components that propagate at the velocity of light, see e. g. Planck collaboration (2020) or Workman (2022).

The density $u_{\Lambda,\text{QED}}$ represents a density of a zero-point energy, ZPE, see e. g. Ballentine (1998).

For the case of the VD, it is shown that the zero-point energy ZPE_ω with a circular frequency ω is minimal energy that an object can have according to the Heisenberg uncertainty relation, see Carmesin (2024b, section 2.5). That derivation does not use the VD. Consequently, that result holds for all quantum objects. In particular, that fact holds for the zero-point energies ZPE_ω that provide the energy density $u_{\Lambda,\text{QED}}$.

However, the classical expectation value of the zero-point energies ZPE_ω and, consequently, of the energy density $u_{\Lambda,\text{QED}}$ are zero:

$$\langle u_{\Lambda,\text{QED}} \rangle_{\text{classical}} = 0 \qquad \{23\}$$

As a consequence, that energy density does not contribute to the density ρ_r in Eq. {1}.

The above Eq. {23} can also be derived as follows: The VPs can form mass in the process of a phase transition, see Higgs (1964), Carmesin (2021a-b). Similarly, the VPs cause the elementary charge as well as the electromagnetic field that are generated by an elementary charge, for instance, see Carmesin (2021c, 2022b). Moreover, a VP exhibits an energy density of its gravitational field and a generalized kinetic energy density, whereby the sum of both, the complete energy density, is zero, see (Carmesin 2021a, 2023a, 2024a-d). This property is not changed during the above phase transitions, so that Eq. {23} holds in the following form:

$$u_{\Lambda,\text{QED,complete}} = 0 \qquad \{24\}$$

2.2.2. Didactic analysis

In a first didactic step, it is explained with help of topics derived in advance, why the classical energy of a ZPE is zero, and why the complete energy of a ZPE of electromagnetic radiation is zero. As no derivation is required, there is no special learning barrier in this step.

2.2.3. $u_{\Lambda,\text{QED}}$ does not provide LFV or GFV

The complete energy density $u_{\Lambda,\text{QED,complete}}$ of the zero – point oscillations of the electromagnetic field is zero, see Eq. {24}, and $u_{\Lambda,\text{QED,complete}}$ includes no available energy ($n_{\mu'} \geq 1$), that would be available for a transformation. Consequently, $u_{\Lambda,\text{QED,complete}}$ does not provide any LFV or GFV. For comparison, the dynamic density $\rho_{r,0}$ includes available energy ($n_{\mu'} \geq 1$) and contributes to the (homogeneous, see Carmesin (2023a,2024d,g)) dynamic density of the universe, and it provides GFV. For comparison, the VD provides the energy density of volume u_{vol} in accordance with observation, see Carmesin (2024c,g).

2.2.4. Physical analysis

(1) Dynamic density of volume in Eq. {1}:

In Eq. {1}, the density ρ_Λ contributes to the squared Hubble rate H^2, and Einstein defined ρ_Λ by that measurable contribution. Carmesin (2023a, 2024c-d) showed that the main part of the observed value $\rho_{\Lambda,\text{obs}}$ is the dynamic density of the volume in nature, ρ_{vol}.

(2) ρ_{vol} causes local formation of volume:

The density ρ_{vol} provides the local formation of volume, LFV, see the law of locally formed volume, LFV in Carmesin (2023a, 2024a-d). The squared rate of the LFV in a direction $\boldsymbol{j}$ is proportional to the field as follows, see Carmesin (2024a):

$$\underline{\dot{\varepsilon}}_{L,jj}^2 c^2 = G_{gen,j}^2 \qquad \{25\}$$

Hereby, $G_{gen,j}$ is the component j of the generalized field.

(3) How ρ_{vol} causes the squared field:

Firstly, we provide a semiclassical description: A VP of minimal energy causes field $\vec{G}_{gen}$ in its vicinity, corresponding to a gradient of relative additional volume. In this manner, that VP generates LFV.

Secondly, we provide a description of the process of LFV at the level of ladder operators:

A zero-point energy ZPE_ω of a volume-portion, VP, causes a nonzero squared field according to the law of the nonzero squared field in Carmesin (2024d): The squared field exhibits the following nonzero expectation value:

$$\langle n_{\mu,} | \vec{G}_{gen}^2 | n_{\mu'} \rangle = G \int d\mu \hbar\omega_\mu \left(n_\mu + \tfrac{1}{2}\right) \delta_{\mu\mu'} \qquad \{26\}$$

The matrix element $\langle n_\mu | \vec{G}_{gen}^2 | n_{\mu'} \rangle$ in the above Eq. can be expressed by the ladder operators, see Carmesin (2024d):

$$\langle n_\mu | \vec{G}_{gen}^2 | n_{\mu'} \rangle = \frac{\hbar c^2 G}{2\omega_\mu} \int d\mu \int d\mu' \vec{k}_\mu \vec{k}_{\mu'} f_\mu f_{\mu'}^{cc} M_{\mu\mu'} \{27\}$$

$$M_{\mu\mu'} := \langle n_\mu \left| \left(a_\mu^+ + a_\mu\right) \left(a_{\mu'}^+ + a_{\mu'}\right) \right| n_{\mu'} \rangle \qquad \{28\}$$

In this manner, the VPs of minimal energy, cause other VPs of minimal energy.

(4) States of the ZPE_ω that provide $u_{\Lambda,\text{QED}}$:

The ladder operators in Eq. {28} act upon states of volume-portions, see Carmesin (2024b). Consequently, the application of such a ladder operator upon a state of the electromagnetic field provides zero as a result:

$$a_\mu |0_{\mu,p}\rangle = 0 \;\; \& \;\; a_\mu^+ |0_{\mu,p}\rangle = 0 \qquad \{29\}$$

Consequently, the states $|0_{\mu,p}\rangle$ do not cause LFV.

As a consequence of sections (2.2) and (2.3), the zero-point energy of the electromagnetic field does not provide the formation of additional volume in Eq. {1}. Thus, the zero-point energy of the electromagnetic field does not contribute to the squared Hubble parameter in Eq. {1}. Consequently, the zero-point energy of the electromagnetic field does not contribute to the measured value $\rho_{\Lambda,obs}$, as Einstein (1917) defined $\rho_{\Lambda,obs}$ by its effect upon the Hubble rate in Eq. {1}.

However, the zero-point energy of the electromagnetic field has no effect upon the Hubble parameter, so that ZPE is not part of $\rho_{\Lambda,obs}$. Thus, that ZPE is no part of ρ_Λ, as that density relies on the observed value $\rho_{\Lambda,obs}$.

We summarize our result:

Theorem: The electromagnetic ZPE forms no volume.

(1) As the electromagnetic ZPE has no classical energy, it is not part of ρ_r in Eq. {1}. So that ZPE does not form volume via ρ_r.

(2) As the ladder operators of the VD applied to a state of electromagnetic ZPE $|0_{\mu,p}\rangle$ provides zero, that state $|0_{\mu,p}\rangle$ does not cause LFV. Thus, that state does not contribute to ρ_Λ by the process of LFV.

(3) Consequently, the electromagnetic ZPE does not contribute to the Hubble rate in Eq. {1}. Accordingly, the electromagnetic ZPE is presumably compensated by a corresponding negative energy. Similarly, the kinetic energy of the ZPE of volume is compensated by a negative energy.

(4) This explains why the energy density u_{vol} is part of u_Λ, but the energy density $u_{\Lambda,\mathrm{QED}}$ is not part of u_Λ.

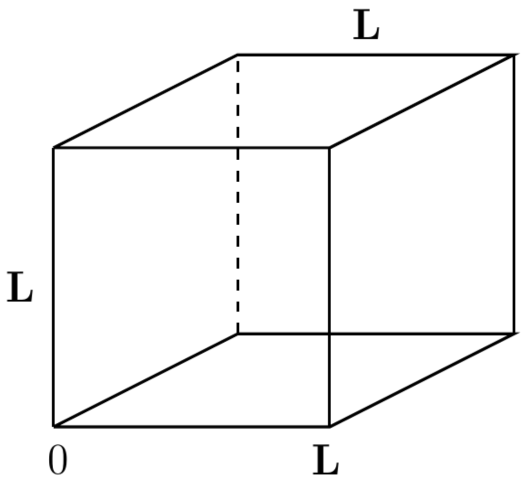

Fig. 2: A cube with length L of the edges is used in order to derive the energy density $u_{\Lambda,\mathrm{QED}}$.

2.3. Energy density of electromagnetic ZPOs

2.3.1. Physical analysis

The energy density $u_{\Lambda,\mathrm{QED}}$ corresponds to a vanishing complete energy density, see Eq. {24}. Consequently, all possible modes can form, up to a maximal wave vector k_{max}. The integral of these modes in a cube of length L, see Fig. (2), provides the energy density, see e. g. Ballentine (1998), Carmesin (2020a):

$$u_{\Lambda,\mathrm{QED}} = \frac{\hbar c \cdot k_{max}^4}{8\pi^2} \qquad \{30\}$$

The largest possible value of k_{max} is provided with help of the Planck length $L_P = 1.616 \cdot 10^{-35} m$. If a ball with the radius of one L_P is at each corner of the cube in Fig. (2), then $L = 2L_P$, and $k_{max} = \frac{\pi}{L}$. In that case, the energy density is as follows:

$$u_{\Lambda,\mathrm{QED}} = \frac{\hbar c \cdot k_{max}^4}{8\pi^2} = 3.6 \cdot 10^{112} \frac{J}{m^3} \qquad \{31\}$$

2.3.2. Didactic analysis

In a first didactic step, the maximal possibly energy density is calculated. This step has no special learning barrier, as the equation is taken from the literature. This step is valuable, as the students become competent in analysing essential lengths and energy densities on their own.

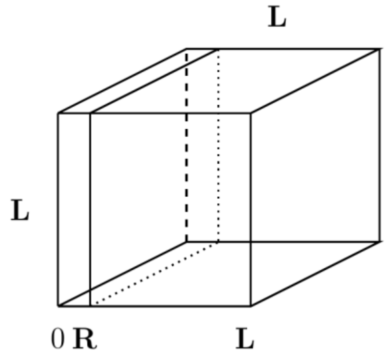

Fig. 3: A cube with length L of the edges, and with a plate at R and parallel to the faces of the cube. Two parallel conducting plates at $x = 0$ and $x = R$, each with area L^2 attract each with the Casimir force $F_{Casimir} = \frac{\hbar c \pi^2 L^2}{240 R^4}$.

2.4. Observed Casimir force

2.4.1. Physical analysis

The zero-point oscillations of the electromagnetic field are reflected at two parallel electrically conducting plates, see Fig. (3). Thereby, a momentum transfer takes place, and a force is exerted upon the plates, see Casimir (1948), Ballentine (1998). Schmidt et al. (2022) observed such forces at plates with a distance of $h = 100\ nm$. The corresponding value of k_{max} is as follows:

$$k_{max} = \frac{\pi}{100\ nm} = 3.14 \cdot 10^7 \frac{1}{m} \qquad \{32\}$$

With it, the corresponding energy density is as follows:

$$u_{\Lambda,\mathrm{QED}} = \frac{\hbar c \cdot k_{max}^4}{8\pi^2} = 390.3 \frac{J}{m^3} \qquad \{33\}$$

Thus, the observed value of $u_{\Lambda,\mathrm{QED}}$ is clearly larger that the observed value of the energy density of the cosmological constant:

$$\mathrm{u}_{\Lambda,obs} = 5.133\ (\pm 0.243) \cdot 10^{-10} \frac{\mathrm{J}}{\mathrm{m}^3} \qquad \{34\}$$

2.4.2. **Didactic analysis**

In a didactic step, the energy densities in Eqs. {30-34} are derived. Thereby, the comparison of Eqs. {3,33} and {34} provides a cognitive conflict, the cosmological constant problem.

This step is insightful, as it shows that values of $u_{\Lambda,\mathrm{QED}}$, that are based on observation, are far beyond the value of the energy density that has been observed at intergalactic space, $u_{\Lambda,obs}$.

2.5. **Explanation of the Casimir force**

2.5.1. **Physical analysis**

(1) The Casimir force in Fig. (3) is explained by the transfer of momentum. Thereby, the momentum corresponds to the kinetic energy: $E_{kin} = p \cdot c$. Consequently, this momentum transfer takes place, irrespective of the value of the complete energy density:

$$\frac{\hbar c \pi^2 L^2}{240 R^4} = F_{Casismir} = F_{reflction} = \frac{\Delta p}{\Delta t} \qquad \{35\}$$

That force is derived from Eq. {30}, see Carmesin (2024g).

2.5.2. **Didactic analysis**

In a first didactic step, the Casimir force is derived and confirmed by observations, see Ballentine (1998), in order to show that zero – point oscillations of the electromagnetic field are measurable.

In a second didactic step, the complete energy density $u_{\Lambda,\mathrm{QED,complete}} = 0$ in Eq. {24} is derived. Thus, the VD clarifies, why the electromagnetic zero – point oscillations have zero complete energy density.

In a third didactic step, the students realize that the derived energy density of volume u_{vol} is in accordance with observation. Thus, the VD clarifies, how the observed energy density of volume and outer space is formed. As these results can be derived, there is no special learning barrier in principle, in this step.

3. **Experience: learning process and learners**

The experiences with learning groups have been documented in terms of photographs of the blackboard and with help of additional reports. These are summarized as follows.

The topic has been presented in general studies courses at the university. The learning process was enriched by a permanent discussion of the achieved results and by exercises about the derived relations. In particular, the learning process took place as follows:

In first unit, the concepts of Einstein’s (1917) cosmological constant and of the corresponding dynamics density and energy density are treated. That unit requires 30 minutes, if the dynamics in Eq. {1} have already been introduced. The students like this topic, as the source of the accelerated expansion is very inspiring and insightful.

In a second unit, the observed data are presented and discussed. With it, the CCP becomes evident. The students like this step, as it makes transparent a deep problem of present-day physics. The unit requires 15 minutes.

In a third unit, the ladder operators are introduced in a descriptive manner. Some students know already a fundamental derivation for the case of volume in nature, see Carmesin (2024a-d). The students like that method, as it is very intuitive, and since it clarifies the spectrum. The unit requires 45 to 90 minutes, with variations depending on the depth of explanations or derivations.

In a fourth unit, the results derived for the energy density of volume are summarized. The students like this unit, as the mentioned process of the formation of volume is very insightful, see e. g. Carmesin (2024c-d). This unit requires 30 minutes.

In unit five, the energy density of $u_{\Lambda,\mathrm{QED}}$ is summarized and explained. The students like this unit, as it is very insightful to realize that it is not obvious to derive a complete energy density. The unit requires 20 to 45 minutes.

In unit six, it is clarified, why the VPs cause LFV, but the ZPE of the electromagnetic field does not. At a first semiclassical level, the point is quite evident. At the level of ladder operators, the point is derived in a more formal manner. The students discuss the essential differences. This makes sense, as the differences are fundamental. Indeed, an even more fundamental derivation can be achieved on the basis of a derivation of the elementary charge from VPs, see Carmesin (2021c, 2022b). The unit requires 45 to 90 minutes.

In unit seven, values of $u_{\Lambda,\mathrm{QED}}$ are analysed. The students like such calculations, as they are quite simple and insightful. The unit requires 20 minutes.

In unit eight, values of $u_{\Lambda,\mathrm{QED}}$ are analysed with help of observed Casimir forces. The students like such calculations, as they are very simple and insightful. The unit requires 20 minutes.

In unit nine, the principle underlying the Casimir force is derived. The students like that derivation, as it is very clear, and as the Casimir force provides clear empirical evidence. The unit requires 15 minutes.

A quantum gravity group of a research club meets 90 minutes each week. Thereby topics such as quantum computers, cosmology, astrophysics or quantum gravity are treated. In that group, essentially the same learning process has been treated in 2023. Also in this case, all questions have been discussed directly, and exercises have been performed.

Altogether, in all learning groups, the learners asked questions. These have been discussed directly in a fully sufficient manner. Moreover, exercises have been used in order to achieve sufficient training, metacognitive activity and familiarity with the new concepts. In some of the exercises, the students were instructed so that they were able to achieve parts of the derivations on their own. This is an efficient test of the ability of the students, and it provides self-esteem to the students in a convincing manner.

4. Discussion

Photovoltaics are ubiquitous. More generally, quantum technologies are essential for our everyday life. Casimir forces, for instance, are essential in nanotechnologies, see e. g. Gong et al. (2021) Accordingly, a fundamental problem of physics, the cosmological constant problem CCP, related to such a relevant topic, is interesting and inspiring.

Indeed, we can solve the CCP on the basis of the dynamics of volume in nature. This result is very inspiring, as we solved also other fundamental problems on the basis of that volume dynamics, see e. g. Carmesin (2023a, 2024a-g).

The learning process is based on the hypothetic deductive method, see the section about the epistemology. Such a testing of a hypothesis and such a deduction from prior knowledge have a high learning efficiency, see Hattie (2006). Moreover, the learning process uses everyday life contexts, so that the learning is meaningful, see Muckenfuß (1995) and achieves an additional high learning efficiency, see Hattie (2006). In the particular case, applications to quantum cryptography and quantum computing are very motivating. For more interesting examples, see Carmesin (2020c).

The learning process has been tested in several learning groups. The learning process includes nine units, some of which are quite short. This indicates that many fields of physics are combined or unified. Such use of prior knowledge provides an especially high learning efficiency, see Hattie (2006). This learning process has been tested at university courses as well as in research club courses. In all these learning groups, the students were able to perform exercises and to use instructions in order to derive parts of the theory. Thus, the topic provides a large amount of self-esteem to the learners.

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