Students Analyse the Impact of the H_0 Tension on the Worldview

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Abstract

In everyday life, we are used to space and time. Thereby, it is very effective and clear to organize our life and our knowledge chronologically. Hereby, the Big Bang is the starting point of our chronological knowledge. Thereby, the rate of expansion of space as well as the age t_0 of the universe are derived from an observed value $H_{0,obs}$. It is the observed value of a fundamental key quantity: the Hubble constant H_0. However, there are different observed values $H_{0,obs}$. For instance, the cosmic microwave background CMB provides $t_{0,CMB} = 13.83$ billion years, whereas radiation with the cosmological redshift $z = 0.055$ provides $t_{0,z=0.055} = 12.66$ billion years. Obviously, the difference is essential for our chronological organization of our knowledge about the world. Carmesin (2023a) derived the dynamics of volume in nature, the volume dynamics, VD. They bridge general relativity and quantum physics. In this paper, we use the VD to derive the theoretical values of $H_{0,theo}(z)$ as a function of the cosmological redshift z, in precise accordance with observation. For that derivation, we provide a learning process, so that you can directly use the concept in your courses. The learning process has been tested in various learning groups, and experiences are reported.

1. Introduction

1.1. On Einstein's idea of Λ

The expansion of space can be derived from general relativity, GR, see Einstein (1917), Friedmann (1922) and Lemaître (1927). Thereby, a uniform scaling of space is derived. In general, such a uniform scaling can be described by the time evolution of a scale radius $r(t)$ in Fig. (1): When space expands by a factor q, then r is multiplied by q. That evolution is described by the following differential equation, DEQ:

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_K + \rho_\Lambda) \text{ with } H = \frac{\dot r}{r} \quad \{1\}$$

Hereby, G is the universal constant of gravity, and H is the Hubble parameter. Moreover, four densities are distinguished, so that each density has a characteristic scaling behaviour as a function of the scale radius r:

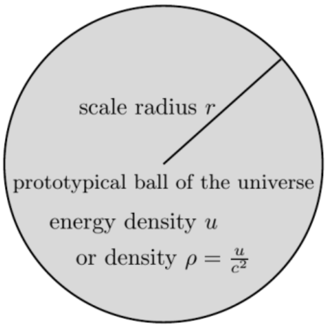

Fig. 1: A prototypical ball of the universe with a scale radius r and an energy density u. The energy density can be expressed in terms of a density or dynamic density $\rho = \frac{u}{c^2}$.

ρ_r is the density of radiation, ρ_m is the density of matter, including cold dark matter, CDM, see Planck collaboration (2020), ρ_K is the density of a curvature parameter, it is zero according to observation, see Planck collaboration (2020), and Carmesin (2023c) proved it, ρ_Λ is the density of the cosmological constant, it does not change as a function of the scale radius r. The model in Eq. {1}, including Λ and CDM, is called ΛCDM cosmology, Workman et al. (2022).

1.2. On the ΛCDM model

Insights about the age t_0 of the universe can be achieved by an analysis of the present-day values of the ΛCDM model. A present-day value of a quantity is marked by the subscript zero. For instance, the present-day value of the time is t_0, see Fig. (2):

$$t_{present-day} =: t_0 \text{ with } t_{Big\,Bang} := 0 \quad \{2\}$$

At value $\rho_K = 0$, the density is called critical density:

$$\rho_{present-day} =: \rho_{cr,0} \quad \{3\}$$

The ratios of the particular densities and the critical density are called density parameter:

$$\Omega_\Lambda := \frac{\rho_\Lambda}{\rho_{cr,0}} \;\&\; \Omega_m := \frac{\rho_m}{\rho_{cr,0}} \;\&\; \Omega_r := \frac{\rho_r}{\rho_{cr,0}}; \quad \{4\}$$

According to the cosmological redshift, the densities in Eq. {1} are functions of the scale radius:

$$H^2 = \frac{8\pi G}{3} \cdot \rho_{cr,0} \cdot \left(\Omega_r \frac{r_0^4}{r^4} + \Omega_m \frac{r_0^3}{r^3} + \Omega_\Lambda\right) \quad \{5\}$$

In the ΛCDM model, the present-day value of the Hubble parameter H is regarded as a constant, named Hubble constant:

$$H(t_0) =: H_{0,\Lambda CDM} = \sqrt{\frac{8\pi G}{3}\rho_{cr,0}} = \frac{1}{t_{H_0}} \quad \{6,7\}$$

Its inverse is called Hubble time t_{H_0}. The present-day time is equal to the Hubble time multiplied by the following integral I_0:

$$t_0 = t_{H_0} \cdot I_0 \;\&\; I_0 = \int_0^1 \frac{x \cdot dx}{\sqrt{\Omega_r + \Omega_m x + \Omega_\Lambda x^4}} \approx 0.95 \quad \{8\}$$

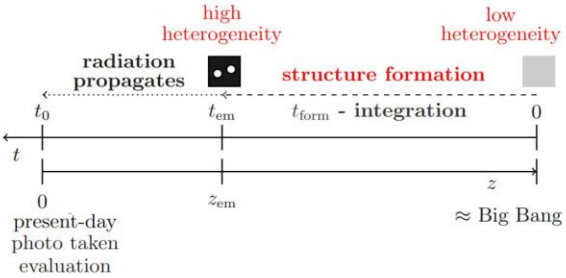

Fig. 2: The time t after the Big Bang and the corresponding cosmological redshift: Heterogeneity or structure in the universe has been evolving since the Big Bang. Structure is observed with help of radiation or objects emitted at a time of emission t_{em}. Such objects can be electromagnetic waves, neutrinos or gravitational waves. So, a photograph of the heterogeneity at t_{em} can be taken.

The age of the universe is a calendar date. Thus, t_0 cannot be derived from universal constants of physics. Instead, t_0 is measured. Hence, the Hubble time t_{H_0} is measured. Thence, H_0 is measured. This is an opportunity to check the ΛCDM model: H_0 can be measured by using physical objects that have been emitted at a time t or a corresponding cosmological redshift z or a scaled time $\tilde{t}$, see e. g. Hobson (2006), Carmesin (2019a):

$$\tilde{t} := \frac{t}{t_{H_0}} = \frac{1}{1+z} \text{ or } \tilde{t}_{em} = \frac{1}{1+z_{em}} \quad \{9\}$$

Thus, in general, the observed values $H_{0,obs}$ of the H_0 form a function of the cosmological redshift:

$$H_{0,obs}(z) = \text{function}(z) \quad \{10\}$$

If that function is a constant, then the ΛCDM model is confirmed. Otherwise, the ΛCDM model is falsified according to the hypothetico-deductive testing, see e. g. Kircher, Girwidz und Häußler (2001, section 4.1.2), Niiniluoto, Sintonen, Wolenski (2004, S. 214). As the level of confidence is above 5 σ, that function is not a constant, see Riess et al. (2022).

1.3. On the observed values of $H_{0,obs}$ and t_0

Using the cosmic microwave background, CMB, emitted at $z_{CMB} = 1090.3$, the Planck collaboration (2020) achieved the following observed value:

$$H_{0,obs}(z = 1090.3) = 66.88\,(\pm 0.92)\ \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \quad \{11\}$$

Hereby, the unit Megaparsec is as follows, see Workman et al. (2022):

$1Mpc = 3.085\,677\,581\,49 \cdot 10^{19}\ km$

Thus, $H_{0,obs} = 2.167\,(\pm 0.03) \cdot 10^{-18}\ \frac{1}{\text{s}}$ {12}

The observed density parameters are as follows, see Planck collaboration (2020) or Carmesin (2019a):

Thereby, $\Omega_\Lambda = 0.679\,(\pm 0.013)$ {13}

Hence, $\Omega_m = 0.321\,(\pm 0.013)$ {14}

Hereby, $\Omega_r = 9.625 \cdot 10^{-5}$ {15}

Thus, $I_0 = 0.9455$ {16}

Thus, the age of the universe is as follows:

$t_{0,CMB} = 13.83\,(\pm 0.24) \cdot 10^9$ years {17}

The density parameters have also been derived from the VD, see Carmesin (2021a). Based on the observation of galaxies at an averaged cosmological redshift $\langle z \rangle = 0.055$, Riess et al. (2022) observed:

$$H_{0,obs} = 73.04\,(\pm 1.01)\ \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \quad \{18\}$$

With it and with Eqs. {13} to {16}, the age of the universe is as follows:

$t_{0,z=0{,}055} = 12.66\,(\pm 0.22) \cdot 10^9$ years {19}

1.4. On the formation of volume in nature

By definition of the cosmological constant, see Einstein (1907), the corresponding observed energy density $u_{\Lambda,\text{obs}} = c^2 \cdot \rho_{\Lambda,\text{obs}}$ is that energy density, that does not change as a function of the scale radius or of the cosmological redshift, see Eq. {5}. For instance, if a measurement A device can measure an energy density u_A, that does not change as a function of the cosmological redshift z, then u_A is equal to $u_{\Lambda,\text{obs}}$ or u_A is a part of $u_{\Lambda,\text{obs}}$. The energy density of volume in nature, $u_{vol} = c^2 \cdot \rho_{vol}$, does not change as a function of the scale radius. Accordingly, u_{vol} is a part of $u_{\Lambda,\text{obs}}$. In the following, u_{vol} is analysed:

When the space expands, then a global formation of volume, GFV, occurs. This is caused by a local formation of volume, LFV. We will analyse how LFV causes GFV, and we will derive the energy density u_{vol} from that process. For it, we will use the dynamics of volume in nature, the volume dynamics, VD, see Carmesin (2024a) or Carmesin (2023a, 2021a). As a first test of that VD, the VD provides the curvature of space in the vicinity of a mass, see Fig. (3).

1.5. Epistemology

Kircher, Girwidz und Häußler (2001, section 4.1.2) describe the hypothetic deductive method. In the epistemological literature, this method is also called hypothetico-deductive testing (Niiniluoto, Sintonen, Wolenski 2004, S. 214). The method consists of three steps: In the hypothetic step, a thesis or hypothesis is suggested for testing. In the deductive step, implications are derived. In the third step, the implications are compared with observation. Hereby, in principle, a falsification should be possible. This method is used here as well as in Carmesin (2024a-g, 2017, 2018a-b, 2019a-b, 2020a-c, 2021a-d, 2022a-c, 2023a-f).

2. Didactic analysis

2.1. On LFV

2.1.1. Physical analysis

The VD have been derived directly from evident properties of volume, this is presented in a parallel paper in this report about the DPG conference in March 2024 in Greifswald, see Carmesin (2024a), or

Carmesin (2023a, 2021a). For the present purpose, the locally formed volume, LFV, is essential, see Carmesin (2023a), Carmesin (2024a, g): For it, the normalized rate $\underline{\dot{\varepsilon}}_L$ of LFV is defined: If the increment of additional volume $\underline{\delta V}$ forms during an increment of time $\underline{\delta\tau}$ in an increment dV_L of volume, then that volume forms at the following normalized rate $\underline{\dot{\varepsilon}}_L$:

$$\underline{\dot{\varepsilon}}_L = \frac{\delta V}{\underline{\delta\tau} \cdot dV_L} \quad \text{, see Fig. (3)} \qquad \{20\}$$

The law of LFV is: At a gravitational field $|\vec{\mathbf{G}}^*|$, there occurs, LFV, at the following normalized rate:

$$\underline{\dot{\varepsilon}}_L = \frac{|\vec{G}^*|}{c} \quad or \quad \underline{\dot{\varepsilon}}_{L,ii} = \frac{|\vec{G}^*|}{c} \qquad \{21\}$$

At a d_{GP} based distance R, a mass M causes the field:

$$\vec{G}^*(\mathrm{R}) = -\frac{GM}{R^2} \cdot \vec{e}_L \text{ with direction vector } \vec{e}_L \quad \{22\}$$

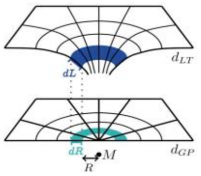

Fig. 3: In the vicinity of a mass M or effective mass M_{eff}, the radial increment dL of the light travel distance d_{LT} is increased with respect to the original increment dR that would occur in the limit M to zero. This increment dR is called gravitational parallax distance d_{GP}, see Carmesin (2023a). Hereby, $dV_L = 4\pi R^2 dL$ and $dV_R = 4\pi R^2 dR$.

2.1.2. **Didactic analysis**

In a didactic step, the relations {20} to {22} are introduced and exercises are performed. A derivation in Carmesin (2023a or 2024a) is used. Thus, this step has no special learning barrier.

2.2. **Introduction of the process of GFV by LFV**

2.2.1. **Physical analysis**

(1) That process is analysed in an especially ideal case, in a universe that consists of volume only.

(2) At a location R_0, a region with the size of a probe volume dV_0 is marked, see Fig. (4). R_0 and dV_0 can be chosen freely. R_0 and dV_0 are constant or fixed during the whole process.

(3) During the time t_0 since the Big Bang until now, the present volume of the universe has been forming. In particular, in that region, the amount dV_0 of volume forms during t_0.

(4) The formation of the volume in dV_0 is caused by dynamic masses dM_j in the universe.

(5) Thus, we will add all increments of LFV that are caused in dV_0 by the dynamic masses dM_j in the universe. Remind that these consists of volume only.

2.2.2. **Didactic analysis**

In a first didactic step, the elements (1) to (4) of the process are introduced with help of Fig. (4). This step has no special learning barrier, as the four elements describe a clear process of formation and propagation of volume according to Eqs. {20} to {22}.

In didactic step two, the plan (5) is developed. This step has no special learning barrier for students familiar with analysis.

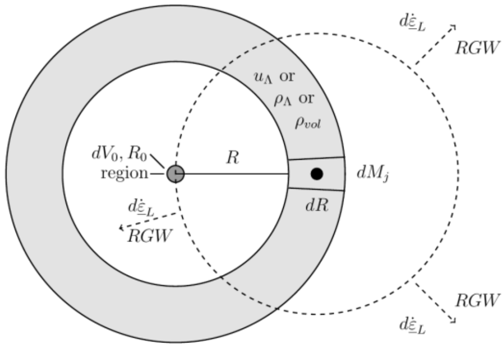

Fig. 4: A dynamic mass dM_j at a distance R from the analysed region (dark grey) with the size dV_0 causes LFV. It propagates in all directions in terms of RGWs, see Carmesin (2023a, 2024a). At a distance R, the mass dM_j causes LFV at a rate $d\underline{\dot{\varepsilon}}_L$.

2.3. **Homogeneous and heterogeneous density**

2.3.1. **Physical analysis**

The density of radiation ρ_r is essential only in the early universe. In the early universe, heterogeneity is negligible, see e. g. Kravtsov and Borgani (2012) or Carmesin (2021a). Thus, it suffices to analyse the homogeneous density of radiation only.

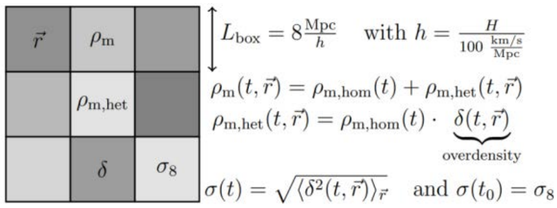

Fig. 5: A dynamic mass dM_j at a distance R from the analysed region (dark grey) with the size dV_0 causes LFV. It propagates in all directions in terms of RGWs, see Carmesin (2023a, 2024a). At a distance R, the mass dM_j causes LFV at a rate $d\underline{\dot{\varepsilon}}_L$.

Heterogeneity is analysed as indicated in Fig. (5):

$$\rho_{m,hom}(t) := \langle \rho_m(t,\vec{r}) \rangle_{\vec{r}} \qquad \{23\}$$

$$\rho_{m,het}(t,\vec{r}) := \rho_m(t,\vec{r}) - \rho_{m,hom}(t) \qquad \{24\}$$

The ratio of the density of heterogeneity and the homogenous density is called overdensity:

$$\delta(t,\vec{r}) := \frac{\rho_{m,het}(t,\vec{r})}{\rho_{m,hom}(t)} = \text{ overdensity} \qquad \{25\}$$

$$\sigma(t) := \sqrt{\langle\delta^2\rangle_{\vec{r}}}\,(t) \qquad \{26\}$$

$$\sigma_8 := \sigma(t_0), \text{ cosmological parameter} \qquad \{27\}$$

2.3.2. **Didactic analysis**

The concept of the observation of heterogeneity is very clear and intuitive, see Fig. (5). So it is presented in one step. Thereby, there occurs one special learning barrier: The length of the box is scaled by the Hubble parameter. This makes sense according to the following analysis at the level of monotonicity: At large values of H, the values of the density ρ are large. Correspondingly, the values of the scale radius are small. Accordingly, the chosen length of the box is small. There is no special learning barrier, as the procedure of the measurement is an arbitrary definition. Thus, nothing has to be derived.

2.4. **Sources of fields and squared fields**

2.4.1. **Physical analysis**

Fields $\vec{G}^*$ can form LFV. For it, we analyse fields and squared fields in various states:
Firstly, we analyse eigenstates $|n_\mu\rangle$ of the number operator and general states, see Carmesin (2024b,g): The matrix element of the generalized field in a general state $|z_\mu\rangle$ is determined as follows:

$$\langle z_\mu|\vec{G}^*|z_\mu\rangle = \sqrt{\frac{\hbar G c^2}{2\omega_\mu}} \int d\mu \vec{k}_\mu \frac{f_\mu}{i} \langle z_\mu|a_\mu^+ + a_\mu|z_\mu\rangle \{28\}$$

Proposition 1: Field of an eigenstate $|n_\mu\rangle$:
In an eigenstate $|n_\mu\rangle$, that matrix element is zero, $\langle n_\mu|\vec{G}^*|n_\mu\rangle = 0$. The proof is in Carmesin (2024g).
Secondly, we analyse coherent states:

$$|z_\mu\rangle = exp\left(-\frac{|z_\mu^2|}{2}\right) \cdot \sum_{n_\mu=0}^{\infty} \frac{z_\mu^{n_\mu}}{\sqrt{n_\mu!}} |n_\mu\rangle \qquad \{29\}$$

Proposition 2: Field of a coherent state $|z_\mu\rangle$:

In a coherent state in Eq. {29}, the expectation value of the field is the following nonzero function:

$$\langle z_\mu|\vec{G}^*|z_\mu\rangle = \sqrt{\frac{\hbar G c^2}{\mathbf{2\omega_\mu}}} \int d\mu \vec{k}_\mu \frac{f_\mu}{i} Re(z_\mu) \langle z_\mu|z_\mu\rangle \{30\}$$

The proof is in Carmesin (2024g).

The field of a coherent state has very small fluctuations. So it can be interpreted as a classical field, see e. g. Ballentine (1998, section 19.4).

Thirdly, we analyse the volume formed by a homogeneous density $\rho_{m,hom}(t)$ of matter, see Eq. {23}. Such a density consists of many small masses m_j. Each such mass causes a field and a rate $\underline{\dot{\varepsilon}_{L,jj}} = \frac{|\vec{G}^*|}{c}$ in its very near vicinity, see Eqs. {21} and {22}. As the masses are part of a homogeneous density $\rho_{m,hom}(t)$, these fields cancel to zero, and the rates $\underline{\dot{\varepsilon}_{L,ii}}$ average, whereby they transform to an isotropic rate $\underline{\dot{\varepsilon}_{L,iso}}$, see Carmesin (2023a):

Theorem 1: Law of the rate of formation of isotropic volume by a homogeneous density:

In a homogeneous system consisting of objects that cause fields with relatively small fluctuations, such as fields of coherent states, the VPs form as follows:

(1) At a microscopic portion of energy or mass, there is a rate $\underline{\dot{\varepsilon}_{jj}}$ of unidirectional formation of volume.

(2) Correspondingly, the object causes a field in its near vicinity.

(3) The fields, caused at many such objects, average to zero. Consequently, the object of the homogeneous system causes no long range field. Thereby, the object forms unidirectional rates $\underline{\dot{\varepsilon}_{jj}}$ in its near vicinity. The unidirectional rates $\underline{\dot{\varepsilon}_{jj}}$ of many objects in the homogeneous system transform to an isotropic rate $\underline{\dot{\varepsilon}_{iso}}$. Thus, such masses of the homogeneous system do not contribute to the rate at the probe volume. Consequently, the energy density u_{vol} in a homogeneous universe is the same as the energy density of volume u_{vol} in an empty universe, see Carmesin (2023a, 2024c). The proof is in Carmesin (2024g).

2.4.2. **Didactic analysis**

In a first didactic step, the field of a state is analysed. For it, the algebra of number operators is applied to the field or generalized field of the VD. Moreover, the field is exact also in curved space. Furthermore, for the case of quantum states including nearly classical coherent states, the field can be evaluated with help of simple algebraic relations. Thus, the field is reliable and exact even for the case of quantum states. Altogether, the field provides the intuition of the gravitational field and the reliability of exact evaluations. Accordingly, there is no special learning barrier in this step, for learners familiar with the algebra of ladder operators in the VD.

In step two, the field of a coherent state is used, Carmesin (2024g), so there is no special learning barrier.

In a third didactic step, the energy density of volume is derived. For it, the cancellation of field vectors is discussed. That process is intuitive, without special learning barrier. The transformation from $\underline{\dot{\varepsilon}_{jj}}$ to an isotropic rate $\underline{\dot{\varepsilon}_{iso}}$ occurs as a result of the averaging of fields and rates $\underline{\dot{\varepsilon}_{jj}}$, see Carmesin (2023a).

2.5. **Volume caused by volume**

The volume caused by LFV has several sources. One of the sources is volume. For it, a possible averaging of generalized gravitational fields is analysed next:

2.5.1. **Physical analysis**

A VP with a circular frequency ω and with a minimal energy E_{min} represents a quantum, and it has the following generalized kinetic energy, see Carmesin (2024b, Eq. {39}): $E_{min}(\omega) = \frac{\hbar\omega}{2}$ {31}

Consequently, the VP is in the number state zero, see Carmesin (2024b, Eq. {72}): $|n_\mu\rangle = 0$ {32}

Proposition 3: Law of the nonzero squared field:

The squared field has a nonzero expectation value:

$$\langle n_\mu|\vec{G}_{gen}^2|n_{\mu'}\rangle = G \int d\mu\, \hbar\omega_\mu \left(n_\mu + \frac{1}{2}\right) \delta_{\mu\mu'} \qquad \{33\}$$

The proof is in Carmesin (2024g). Next, u_{vol} is derived for the ideal case of an empty universe:

Theorem 2: Law of the derived energy density of volume in an empty universe.

In a universe consisting of volume only, the process of GFV from LFV causes the following energy density of pure volume:

$$u_{\Lambda,\text{theo}} = \frac{c^2 H_0^2}{4\pi G} = u_{\text{vol,pure}}, \quad \text{thus,} \qquad \{34\}$$

$\rho_{\text{vol,pure}} = \frac{H_0^2}{4\pi G} = \ 5.600\ (\pm 0.155) \cdot 10^{-27}\ \frac{\text{kg}}{\text{m}^3}$ &
$\Omega_{vol} = \frac{2}{3}$ {35}

We name it the energy density of pure volume:
$u_{\text{vol,pure}} = u_{\text{vol}}$ & $\rho_{\text{vol,pure}} = \rho_{\text{vol}}$
The same result holds for the homogeneous universe.

This result is in precise accordance with observation. The proof is in Carmesin (2024g).

2.5.2. Didactic analysis

In a first didactic step, the squared field is analysed with help of the algebra of ladder operators. Such an analysis has already been analysed in section (2.4). This step has no special learning. In the second step, the energy density of volume is derived with help of the process of GFV by LFV. That derivation has already been analysed in Carmesin (2024c).

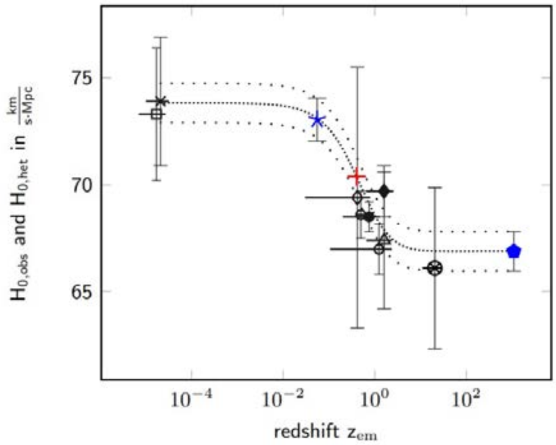

Fig. 6: The Hubble constant as a function of the cosmological redshift. Data points show various measurements. The densely dotted line represents the present theory. Data and theory are in precise accordance. Full circle: Square: Blakeslee (2021). ×: Pesce et al. (2020) and Addison et al. (2018). Star ⋆: Riess et al. (2022). Diamond: Escamilla-Rivera and Najera (2022). Circle: Philcox et al. (2020). Abbott et al. (2020). Full diamond: Cao et al. (2021). Δ: Birrer et al. (2020). ⊗: Cimatti and Moresco (2023). Pentagon: Planck collaboration (2020).

2.6. Volume caused by a heterogeneous density

2.6.1. Physical analysis

Theorem 3: Law of the derived Hubble constant as a function of the cosmological redshift.

(1) In a heterogeneous universe, the process of GFV from LFV causes the following Hubble constant as a function of the cosmological redshift:

$$H_{0,het} = H_{0,\Lambda CDM}\sqrt{\Omega_{\bar{m}} + \Omega_{vol} \cdot (1+\kappa)^{\xi}} \qquad \{47\}$$

Hereby, $H_{0,\Lambda CDM} = \sqrt{\frac{8\pi G}{3} \cdot \rho_{cr.}}$ is the Hubble constant of the ΛCDM model of cosmology. The density parameter $\Omega_{\bar{m}} = \Omega_m + \Omega_r$ is used. Ω_{vol} is the density parameter of the volume. It is essentially equal to the density parameter of the cosmological constant, Ω_Λ. Ω_Λ includes the density of volume and of the volume caused by the heterogeneity. That heterogeneity is the source of the time dependence of the Hubble constant.

(2) Parameters: κ describes the additional rate caused by heterogeneity. The exponent ξ describes the effect of that additional rate upon the Hubble constant. These parameters are determined as follows:

$$\kappa(\tilde{t}_{em}) = \frac{\Omega_{\bar{m}} \cdot \sigma_8 \cdot \tilde{t}_{em}^2}{2\Omega_{\text{vol}}} \text{ and } \xi = \frac{ln(w_+)}{ln(y)} \ \& \ y = 1+\kappa$$

$$\text{with } w_+ = \frac{\Omega_{vol} \cdot y^2}{2} \cdot \left(1 + \sqrt{1 + \frac{4\Omega_{\bar{m}}}{\Omega_{vol}^2 y^2}}\right) \qquad \{48\}$$

(3) Time evolution of the Hubble constant: The time evolution of the Hubble constant is shown in Fig. (6). The derived theoretical results are in precise accordance with observation. Moreover, the theory predicts the full function $H_{0,theo}(z)$. The proof is presented in Carmesin (2024g).

2.6.2. Didactic analysis

In a first didactic step, the concept of the overdensity is applied to a dynamic mass dM_j. This step has no special learning barrier. This step is valuable, as it provides insight into the observation of the time evolution of structure in the universe.

In a second didactic step, the law of LFV is applied to the overdensity of a dynamic mass dM_j. Thereby, squares and standard deviations are derived. This step has no special learning barrier. This step is insightful, as it shows how the VD and the LFV are applied to fluctuations. Moreover, this step is very insightful, as it shows that even fluctuations with a vanishing average cause additional LFV. This is the ultimate source of the Hubble tension.

In a third didactic step, the linear growth theory is applied to the standard deviation derived in part (2). Though the derivation of linear growth theory is quite complex, see Carmesin (2021a), the application of that theory is extremely simple. Hence, this step has no special learning barrier. This step is very valuable, as it shows how a simple law of structure formation is achieved for the case of standard deviations.

In a didactic step (4), the derived rate caused by heterogeneity is related to the rate caused without heterogeneity. Thereby, an approximation in leading order is applied. This step has a small mental learning barrier, as students might think about the reliability of the approximation. As a result, a simple factor κ is derived. Thus, this step has a very clear structure. Thence, this step has no special learning barrier. This step is very valuable and insightful, as it shows how rates can be separated and related to each other.

In a didactic step (5), the effects of the additional rate caused by heterogeneity upon the Hubble constant is derived. The effect is highly nonlinear. Moreover, the effect is mediated by the density ρ_Λ. Thus, this step has a high metacognitive learning barrier. In fact, it is impossible that the students plan a useful and effective treatment on their own. On the other hand, the derivation is exact. Thus, the students achieve a large amount of success in the treatment of this nonlinear and mediated effect. The metacognitive barrier is overcome by clear instructions of all applied steps, see Carmesin (2024g). Moreover, an explicit method of measurement is introduced, so that a clear relation to reality is provided. With it, there is no remaining special learning barrier. This step is very insightful, as it shows that even though the rates and the growth could be analysed in a linear manner, this is not so in the case of the Hubble constant. This is insightful, as the source of that nonlinearity is the fact that the Hubble constant corresponds to the age of the universe, which integrates all growth effects from the Big Bang to the present-day time t_0.

In a didactic step (6), the exponent introduced in part (5) is derived. In principle, this can be achieved numerically. However, a general equation is more useful. The students are provided with the plan that a quadratic equation is achieved with help of appropriate substitutions. With it, the students can derive the exponent on their own. So they can achieve self – esteem in an especially efficient manner.

In didactic steps (7) and (8), the Hubble constant is evaluated for the case of the CMB and $z = 0.055$. These cases are especially valuable, as they show that the local value of the Hubble constant has been achieved at a confidence level above 5 σ. These steps have no special learning barrier.

In didactic steps (9), the derived Hubble constant as a function of the redshift is related to observed values. This step has no special learning barrier. This step is very insightful, as it relates observation with the derived results. This step is the essential step in the hypothetico-deductive testing. Students are encouraged to discuss this testing in a founded and critical manner.

In a didactic step (10), properties of the density of volume are summarized. For it, the properties derived here as well as properties derived with help of the Planck scale are used, see Carmesin (2017, 2018a-b, 2019a-b, 2020a-b, 2021a-b, 2023a, 2024g). As all properties have already been derived, this step has no special learning barrier. This topic is very insightful. Moreover, some learners can connect new and prior knowledge, this has a high learning efficiency, see Hattie (2006).

3. **Experience: learning process and learners**

The experiences with learning groups have been documented in terms of photographs of the blackboard and with help of additional reports. These are summarized as follows.

Since 2021, see Carmesin (2021a), the topic has been presented in several general studies courses at the university. The learning process was enriched by a permanent discussion of the achieved results and by exercises about the derived relations. In particular, the learning process took place as follows:

The law of locally formed volume has been treated in advance. In first unit in section (1), the Hubble tension is presented and the age of the universe is analysed with it. This step is very exciting to the students, as the time evolution of the universe is reanalysed with significant effects. The unit requires 90 minutes, including exercises and discussions.

In a separable second unit, the derivation of $\rho_{\Lambda,\mathrm{theo}}$ has been achieved. For it, the process of GFV by LFV is treated. This includes the derivation of $\rho_{\Lambda,\mathrm{theo}}$ and the law of the derived energy density of volume in an empty universe, as well as the exercises and discussion. This unit provides a large amount of self-esteem, as the students learn how to derive on their own the energy density u_{vol}, corresponding to 67 % of all energy. This unit requires 90 minutes.

In another separable and third unit, the cancellation of fields is analysed. This can be achieved in a semiclassical manner, see Carmesin (2021a, 2023a). That semiclassical treatment requires 30 minutes and is intuitive.

Here, this didactic step is achieved with help of ladder operators applied to the fields of the VD. This step requires 90 minutes. The learners achieve a lot of generalizable competence in this step. Correspondingly, many learners like this algebraic method.

The main unit is section (2.5), the derivation of the time evolution of the Hubble constant. This unit can be presented in 90 minutes in the form of a lecture. The students are very interested in that derivation, after they explored the age of the universe in unit (1), after they derived the dark energy in unit (2), and after they analysed the algebraic structure of fields and their averages in unit (3).

A quantum gravity group of a research club meets 90 minutes each week. Thereby topics such as quantum computers, cosmology, astrophysics or quantum gravity are treated. In that group, essentially the same learning process has been treated in several courses since 2021. Also in this case, all questions have been discussed directly, and exercises have been performed.

Altogether, in all learning groups, the learners asked questions. These have been discussed directly in a fully sufficient manner. Moreover, exercises have been used in order to achieve sufficient training, metacognitive activity and familiarity with the new concepts. In some of the exercises, the students were instructed so that they were able to achieve parts of the derivations on their own. This is an efficient test of the ability of the students, and it provides self-esteem to the students in a convincing manner.

4. **Discussion**

Space is an ubiquitous entity of everyday life. Moreover, time is basic for the chronological organisation of events and knowledge. Accordingly, all essential cultures developed a useful and relatively precise calendar system, see Hoskin (1999).

In modern physics and cosmology, a surprising mystery occurred: the Hubble tension and the local value of the Hubble constant, see Riess et al. (2022). It showed that the traditional ΛCDM model of cosmology cannot explain the observed time evolution of the universe since the Big Bang in a sufficient manner. This exciting mystery is treated in this course in a conclusive, exact and precise manner.

For it, the dynamics of volume in nature are analysed systematically. In fact, that volume-dynamics, VD, explain the essential present-day physical theories: gravity, general relativity and QP. Indeed, this broad and exact basis is essential in order to solve the problem of the Hubble tension. In fact, equipped with the VD, the students can learn how to solve the Hubble tension problem on their own, and how to predict future observations of $H_0(z)$. Thereby, they learn many valuable, insightful and generalizable tools, methods, concepts and theories: beyond the theories that are not able to solve the Hubble tension!

In fact, the VD have been used successfully to solve other problems of fundamental physics. For it, see the other reports about my contributions to the DPG conference in spring 2024 in Greifswald.

Indeed, also the students can apply the VD in order to solve more fundamental problems of physics in the future.

Altogether, we show how the volume of the universe forms in a permanent process. Moreover, we show that the heterogeneity is an additional source for such formation of volume. Indeed, we show in a precise manner, that this heterogeneity is the source of the Hubble tension. Moreover, we derive an advanced value for the age of the universe. This is a gradual impact of the Hubble tension on the world view. Moreover, the Hubble tension indicates that volume in nature forms permanently and locally at masses, energies, volume portions and even at fluctuations – and that volume in nature flows at the velocity of light, whereas we and other masses move slowly.

The learning process is based on the hypothetic deductive method, see the section about the epistemology. Such a testing of a hypothesis and such a deduction from prior knowledge have a high learning efficiency, see Hattie (2006). Moreover, the learning process uses everyday life contexts, so that the learning is meaningful, see Muckenfuß (1995) and achieves an additional high learning efficiency, see Hattie (2006). In the particular case, applications to quantum cryptography and quantum computing are very motivating. For more interesting examples, see Carmesin (2020c).

The learning process of the process of GFV by LFV has been tested in several learning groups. The learning process includes four units with 90 minutes each. This has been tested at university courses as well as in research club courses. In all these learning groups, the students were able to perform exercises and to use instructions in order to derive parts of the theory. Thus, the topic provides a large amount of self-esteem to the learners.

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