Students Learn to Derive the Energy Density of Volume

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Abstract

In everyday life, we are used to the space in which we live. However, physicists need to know or assume properties about that space: Newton (1668) preferred a flat and constant space. Maxwell (1865) proposed that space should be an electromagnetic aether. Einstein (1917) suggested that space would be characterized by a cosmological constant Λ, corresponding to an energy density u_{vol} of space or of volume. Moreover, he provided an equation that determines how u_{vol} is related to the expansion of space since the Big Bang. Based on that relation, Perlmutter et al. (1998) discovered a nonzero observed value $u_{vol,obs}$ of u_{vol}. The derivation of u_{vol} represents an essential and exciting problem about nature and physics. Carmesin (2023a) derived the dynamics of volume in nature, the volume dynamics, VD. They bridge general relativity and quantum physics. In this paper, we use the VD to derive u_{vol}. For that derivation, we provide a learning process, so that you can directly use the concept in your courses. The learning process has been tested in various learning groups, and experiences are reported.

1. Introduction

1.1. On Einstein's idea of Λ

The expansion of space can be derived from general relativity, see Einstein (1917), Friedmann (1922) and Lemaître (1927). Thereby, a uniform scaling of space is derived. In general, such a uniform scaling can be described by the time evolution of a scale radius $r(t)$, see Fig. (1): If space expands by a factor q, then r is multiplied by q. That time evolution can be described by this differential equation, DEQ:

$$\frac{\dot{r}^2}{r^2} = \frac{8\pi G}{3} \cdot (\rho_r + \rho_m + \rho_K + \rho_\Lambda) \qquad \{1\}$$

Hereby, G is the universal constant of gravity. Moreover, four densities are distinguished, so that each density has a characteristic scaling behaviour as a function of the scale radius r:

ρ_r is the density of radiation,

ρ_m is the density of matter,

ρ_K is the density of a curvature parameter, it is zero according to observation, see Planck collaboration (2020), and as a result of a proof, see Carmesin (2023c),

ρ_Λ is the density of the cosmological constant, it does not change as a function of the scale radius r.

A present-day value of a quantity is marked by the subscript zero. Next, the densities in Eq. {1} are expressed as functions of the scale radius:

$$\frac{\dot{r}^2}{r^2} = \frac{8\pi G}{3} \cdot \left(\rho_{r,0}\frac{r_0^4}{r^4} + \rho_{m,0}\frac{r_0^3}{r^3} + \rho_\Lambda\right) \qquad \{2\}$$

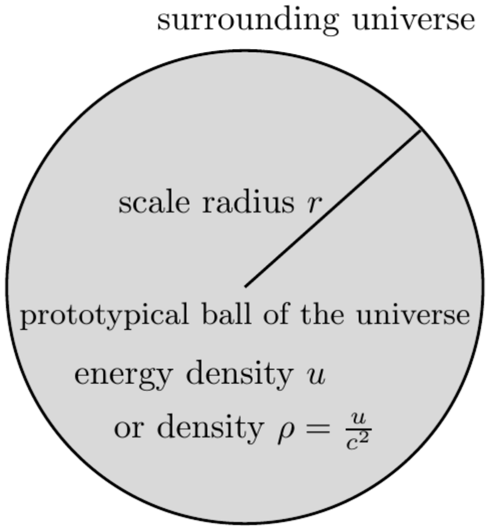

Fig. 1: A prototypical ball of the universe with a scale radius r and an energy density u. The energy density can be expressed in terms of a density or dynamic density $\rho = \frac{u}{c^2}$.

When ρ_Λ becomes essential, r is very large, so that ρ_r becomes very small, so we neglect it in section (1.1). We multiply by r^2 and apply the time derivative:

$$\frac{\partial}{\partial t}\dot{r}^2 = \frac{8\pi G}{3} \cdot \frac{\partial}{\partial t}\left(\rho_{m,0}\frac{r_0^3}{r^1} + \rho_\Lambda r^2\right) \qquad \{3\}$$

$$2\dot{r}\,\ddot{r} = \frac{8\pi G}{3}\left(-\rho_{m,0}\frac{r_0^3}{r^2} + 2\rho_\Lambda r\right)\dot{r} \qquad \{4\}$$

In order to obtain a relative acceleration $\frac{\ddot{r}}{r}$, we divide by $2r\dot{r}$:

$$\frac{\ddot{r}}{r} = \frac{8\pi G}{3}\left(-\frac{1}{2}\rho_m + \rho_\Lambda\right) \qquad \{5\}$$

Einstein (1917) had the idea of a static universe: If the ρ_Λ compensates $\frac{1}{2}\rho_m$ in the above DEQ, then r is not accelerated. Thus, if $\dot{r}$ is zero initially, then $\dot{r}$ remains zero and the universe is static.

For this purpose of a possibly static universe, Einstein (1917) proposed the cosmological constant Λ, corresponding to the density $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$ and energy density $u_\Lambda = \frac{\Lambda c^4}{8\pi G}$, see e. g. Hobson (2006, section 15.1).

1.2. Epistemology

Kircher, Girwidz und Häußler (2001, section 4.1.2) describe the hypothetic deductive method. In the epistemological literature, this method is also called hypothetico-deductive testing (Niiniluoto, Sintonen, Wolenski 2004, S. 214). The method consists of three steps: In the hypothetic step, a thesis or hypothesis is suggested for testing. In the deductive step, implications are derived. In the third step, the implications are compared with observation. Hereby, in principle, a falsification should be possible. This method is used here as well as in Carmesin (2024a-g, 2017, 2018a-b, 2019a-b, 2020a-c, 2021a-d, 2022a-c, 2023a-f).

1.3. On the observed value Λ_{obs}

As a consequence of Eq. {5}, it was clear how ρ_Λ could be measured: If an observer would measure an accelerated expansion of space, then this could be explained by the dynamic density ρ_Λ. In fact, Perlmutter et al. (1998) discovered the accelerated expansion of the universe.

Meanwhile, many observers confirmed the accelerated expansion of the universe. An especially precise measurement of ρ_Λ has been achieved with help of the cosmic microwave background, CMB, see Planck collaboration (2020). That group applied several evaluation procedures, whereby the so-called temperature-temperature correlation is especially robust and used here:

The Hubble constant H_0 is the present-day value of the Hubble parameter $H = \frac{\dot{r}}{r}$, the observed value is:

$H_{0,obs} = 66.88\ (\pm 0.92)\ \frac{\text{km}}{\text{s}\cdot\text{Mpc}}$ with

$1Mpc = 3.086 \cdot 10^{19}\ km,$ thus,

$$H_{0,obs} = 2.167\ (\pm 0.03) \cdot 10^{-18}\ \frac{1}{\text{s}} \qquad \{6\}$$

With it, the so-called critical density is as follows:

$$\rho_{cr.} = \frac{3H_0^2}{8\pi G} = 8.4 \cdot 10^{-27}\ \frac{\text{kg}}{\text{m}^3} \qquad \{7\}$$

The density divided by the critical density is the density parameter, $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$. Its observed value is:

$$\Omega_{\Lambda,obs} = 0.679\ (\pm 0.013) \qquad \{8\}$$

Note that this density parameter means that 67.9 % of all energy and matter in the universe is the energy of ρ_Λ, the so-called dark energy, see Huterer (1999), Planck collaboration (2020), Workman et al. (2022).

Thus, the observed value of ρ_Λ is:

$$\rho_{\Lambda,obs} = \Omega_{\Lambda,obs} \cdot \rho_{cr.} = 5.704\ (\pm 0.27) \cdot 10^{-27}\ \frac{\text{kg}}{\text{m}^3} \qquad \{9\}$$

1.4. On the formation of volume in nature

The energy density $u_\Lambda = c^2 \cdot \rho_\Lambda$ includes all physical energy densities that do not change as a function of the scale radius or of the cosmological redshift, see Eq. {2}. The energy density of volume in nature, $u_{vol} = c^2 \cdot \rho_{vol}$, does not change as a function of the scale radius. Accordingly, u_{vol} is analysed next:

When the space expands, then a global formation of volume, GFV, takes place.

This is caused by a local formation of volume, LFV.

We will analyse this process of LFV causing GFV, and we will derive the energy density $\boldsymbol{u_{vol}}$ from that process.

For it, we will use the dynamics of volume in nature, the volume dynamics, VD, see Carmesin (2024a, 2023a, 2021a).

As a first test of that VD, the VD provides the curvature of space in the vicinity of a mass, see Fig. (2).

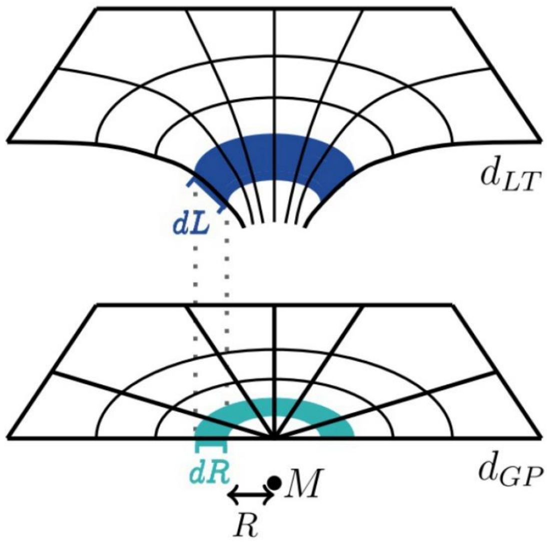

Fig. 2: In the vicinity of a mass M or effective mass M_{eff}, the radial increment dL of the light travel distance d_{LT} is increased with respect to the original increment dR that would occur in the limit M to zero. This increment dR is called gravitational parallax distance d_{GP}, see Carmesin (2023a).

1.5. Organization of the paper

A didactic analysis including a professional analysis is provided in section 2. The learning process including experiences with learning groups are shown in part 3. We discuss our findings in section 4. Of course, there are additional questions about u_Λ that are not treated in this paper. Many of these additional questions are clarified in my parallel papers in the report about the DPG conference in March 2024 in Greifswald, see Carmesin (2024a-g).

2. Didactic analysis

2.1. On LFV

2.1.1. **Physical analysis**

The dynamics of volume in nature have been derived directly from evident properties of volume, this is presented in a parallel paper in this report about the DPG conference in March 2024 in Greifswald, see Carmesin (2024a). Moreover, it has been derived with help of selected results of general relativity, see Carmesin (2023a, 2021a).

For the present purpose, the local formation of volume is essential, see Carmesin (2023a) or Carmesin (2024a, f):

For it, the normalized rate $\underline{\dot{\varepsilon}}_L$ of locally formed volume is defined:

If the increment of additional volume $\underline{\delta}V$ forms during an increment of time $\underline{\delta}\tau$ in an increment dV_L of volume, then that volume forms at the following normalized rate $\underline{\dot{\varepsilon}}_L$:

$$\underline{\dot{\varepsilon}}_L = \frac{\underline{\delta}V}{\underline{\delta}\tau \cdot dV_L} \qquad \{10\}$$

With it, the law of locally formed volume is presented:

At a gravitational field $|\vec{G}^*|$, there occurs locally formed volume, LFV, at the following normalized rate:

$$\underline{\dot{\varepsilon}}_L = \frac{|\vec{G}^*|}{c} \qquad \{11\}$$

Thereby, a mass M causes the following gravitational field at a d_{GP} based distance R:

$$\vec{G}^*(R) = -\frac{GM}{R^2} \cdot \vec{e}_L \qquad \{12\}$$

Hereby, $\vec{e}_L$ is the direction vector in the radial direction.

In the present investigation, the space is globally flat, so that the above Eqs. {11} and {12} hold in an exact manner. For approximate and exact relations in curved space, see Carmesin (2023a, 2024a).

2.1.2. **Didactic analysis**

In a first didactic step, the relations {10} to {12} are introduced and exercises are performed. A possible derivation is presented in Carmesin (2024a). This step has no special learning barrier, as only the application of the relations is required.

2.2. **Introduction of the process of GFV by LFV**

2.2.1. **Physical analysis**

(1) That process is analysed in an especially ideal case, in a universe that consists of volume only.

(2) At a location R_0, a region with the size of a probe volume dV_0 is marked, see Fig. (3). Thereby, the location R_0 and the size dV_0 of the region can be chosen arbitrarily. Hereby, the location R_0 and the region are constant or fixed during the whole process.

(3) During the time t_0 since the Big Bang until now, the present volume of the universe forms. In particular, in that region, the amount dV_0 of volume forms during t_0.

(4) The formation of the volume in that region is caused by all dynamic masses dM_j of the volume in the universe.

(5) As a consequence, we will add or integrate all increments of LFV that are caused in the marked region by all dynamic masses dM_j in the universe. Remind that these consist of volume only.

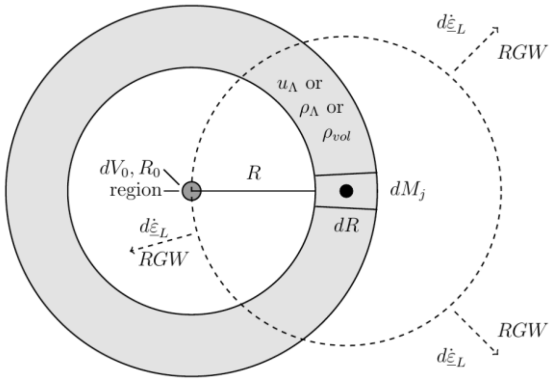

Fig. 3: A dynamic mass dM_j at a distance R from the analysed region (dark grey) with the size dV_0 causes LFV. It propagates in all directions in terms of RGWs, see Carmesin (2023a, 2024a). At a distance R, the mass dM_j causes LFV at a rate $d\underline{\dot{\varepsilon}}_L$.

2.2.2. **Didactic analysis**

In a first didactic step, the elements (1) to (4) of the process of GFV by LFV are introduced with help of Fig. (3). This step has no special learning barrier, as the four elements describe a clear process of formation and propagation of volume according to Eqs. {10} to {12}.

In a second didactic step, the plan (5) is developed. This step has no special learning barrier for students familiar with analysis. The reason is that the adding or integrating is planned for all increments of volume that form in the marked region. That is a clear analytic and arithmetic procedure.

2.3. **Integrating the increments of the process of GFV by LFV**

2.3.1. **Physical analysis**

(1) The dynamic masses dM_j in a shell with centre at R_0, radius R and thickness dR are analysed, see Fig. (3). For each infinitesimal increment dR, the volume of that shell is the product of the surface $4\pi R^2$ and the thickness dR:

$$dV = 4\pi R^2 \cdot dR \qquad \{13\}$$

The dynamic mass dM in that shell is equal to the product of the volume and the density:

$$dM = dV \cdot \rho_\Lambda = 4\pi R^2 \cdot \rho_\Lambda \cdot dR \qquad \{14\}$$

That mass causes the following rate in the marked region, see Eqs. {11} and {12}:

$$d\underline{\dot{\varepsilon}}_L = \frac{G \cdot dM}{R^2 c} = \frac{G \cdot 4\pi R^2 \cdot \rho_\Lambda \cdot dR}{R^2 c} = \frac{G \cdot 4\pi \cdot \rho_\Lambda \cdot dR}{c} \qquad \{15\}$$

(2) We realize that the normalized rate $d\underline{\dot{\varepsilon}}_L$ does not depend on the value of the radius of the shell. Thus, the integration is not complicated.

(3) As the process is introduced by the time from the Big Bang until the present-day, we substitute $dR = c \cdot dt$, since the RGWs propagate with the velocity c:

$$d\underline{\dot{\varepsilon}}_L = 4\pi G \rho_\Lambda dt \qquad \{16\}$$

(4) The integration ranges from $t = 0$ to $t = t_0$. Thus, the integration yields the following normalized rate $\underline{\dot{\varepsilon}}_L$ arriving at the marked region during the time interval $[0, t_0]$:

$$\underline{\dot{\varepsilon}}_L([0,t_0]) = 4\pi G \rho_\Lambda \int_0^{t_0} dt = 4\pi G \rho_\Lambda t_0 \qquad \{17\}$$

(5) The present-day time t_0 is essentially the same as the Hubble time $t_{H_0} = \frac{1}{H_0}$, see e. g. Hobson (2006), Carmesin (2019a):

$$\underline{\dot{\varepsilon}}_L([0,t_0]) = 4\pi G \rho_\Lambda \frac{1}{H_0} \qquad \{18\}$$

2.3.2. Didactic analysis

In a first didactic step (1), the shell is analysed in Eqs. {13} to {15}. This step has no special learning barrier.

In a second didactic step, the integral is introduced and evaluated in Eqs. {16} to {18}. This step has no special mathematical learning barrier for learners familiar with analysis. However, that step has a mental learning barrier for learners that think the universe was small at the Big Bang. This mental barrier is clarified by the fact that the universe has already been infinite at the Big Bang, as the universe is globally flat, see Planck collaboration (2020) or Carmesin (2023c).

2.4. Integrating the increments of the process of GFV by LFV

2.4.1. Physical analysis

(1) The definition of the normalized rate (see Eq. {10}) is applied to the integrated rate in Eq. {18}:

$$\underline{\dot{\varepsilon}}_L([0,t_0]) = \frac{\delta V}{\delta\tau \cdot dV_L} = 4\pi G \rho_\Lambda \frac{1}{H_0} \qquad \{19\}$$

According to the process in section (2.2), we analyse the formation of volume in the marked region with the size dV_0. Consequently, the volume dV_L in the above Eq. is equal to dV_0.

Moreover, in that process, the formed volume $\underline{\delta V}$ in the above Eq. is equal to the size dV_0 of the marked region.

Furthermore, the process takes place during the time from the Big Bang until the present-day time t_0 or $t_{H_0} = \frac{1}{H_0}$.

These three relations are inserted in the above Eq. {19}:

$$\underline{\dot{\varepsilon}}_L([0,t_0]) = \frac{H_0 \cdot dV_0}{dV_0} = H_0 = 4\pi G \rho_\Lambda \frac{1}{H_0} \qquad \{20\}$$

The above Eq. is solved for ρ_Λ. The derived value is the theoretical value and marked by the subscript theo:

$$\rho_{\Lambda,\text{theo}} = \frac{H_0^2}{4\pi G} \quad \text{and} \quad u_{\Lambda,\text{theo}} = \frac{c^2 H_0^2}{4\pi G} \qquad \{21\}$$

This density describes the value of ρ_Λ that is derived by the process of GFV by LFV. It is tested by comparison with observed values next:

(2) The observed value $H_{0,obs}$ in Eq. {6} is inserted:

$$\rho_{\Lambda,\text{theo}} = \frac{H_0^2}{4\pi G} = 5.600\,(\pm 0.155) \cdot 10^{-27}\,\frac{\text{kg}}{\text{m}^3} \quad \{22\}$$

Consequently, the above derived result is in precise accordance with the observed value $\rho_{\Lambda,obs} = 5.704\,(\pm 0.27) \cdot 10^{-27}\,\frac{\text{kg}}{\text{m}^3}$ in Eq. {9}, within the error of measurement.

2.4.2. Didactic analysis

In a first didactic step (1), the conditions of the process of GFV by LFV is applied to the rate in Eq. {18}. The resulting Eq. is solved for the density ρ_Λ in Eq. {21}. This step has no special learning barrier.

2.5. Interpretation of the process of GFV by LFV in a universe consisting of volume only

2.5.1. Physical analysis

(1) The analysed universe consists of volume only. Moreover, it is globally flat, see Planck collaboration (2020) or Carmesin (2023c). Thus, that universe is infinite all time, and the density is the same all time. As a consequence, the derived density of volume $\rho_{\Lambda,\text{theo}}$ is the same all time:

$$\rho_{\Lambda,\text{theo}} = \text{constant} \qquad \{23\}$$

(2) Einstein (1917) defined the density ρ_Λ of the cosmological constant Λ indirectly by the rate of the expansion of the universe, see Eq. {1}. In that definition, ρ_Λ subsumes all physical densities that do not change as a function of the scale radius, see Eq. {2}. The theoretical value derived here consists of volume only, by construction of the process of GFV by LFV in section (2.2). Accordingly, the derived density is definitely the density of volume, without any conceivable additional component that is independent of the scale radius:

$$\rho_{\Lambda,\text{theo}} = \rho_{vol,theo} \qquad \{24\}$$

(3) If the process of GFV by LFV is used at any time t_1 of the universe, different from the present-day time t_0, than Eq. {17} changes to the following Eq.:

$$\underline{\dot{\varepsilon}}_L([0,t_1]) = 4\pi G \cdot \rho_{vol,theo} \int_0^{t_1} dt = 4\pi G \cdot \rho_{vol,theo} \cdot t_1 \qquad \{25\}$$

In particular, the ratio of the rate and the time is a constant, as ρ_Λ is constant in the process of GFV by LFV:

$$\frac{\underline{\dot{\varepsilon}}_L([0,t_1])}{t_1} = \frac{\underline{\dot{\varepsilon}}_L([0,t_0])}{t_0} = 4\pi G \rho_{vol,theo} = \text{constant} \; \{26\}$$

The results are summarized:

Theorem: Law of the derived energy density of volume in an empty universe.

In a universe consisting of volume only, the process of GFV from LFV causes the following energy density of volume:

$$u_{\Lambda,\text{theo}} = \frac{c^2 H_0^2}{4\pi G} = u_{\text{vol}}, \quad \text{thus,} \qquad \{26\}$$

$$\rho_{\Lambda,\text{theo}} = \frac{H_0^2}{4\pi G} = 5.600\ (\pm 0.155) \cdot 10^{-27} \frac{\text{kg}}{\text{m}^3}\ \{27\}$$

The density is a consequence of the process of formation of volume since the Big Bang until the present-day time t_0.

If that process ranges from the Big Bang to another time $t_1 \neq t_0$, then that process provides the same density of volume.

This result is in precise accordance with observation.

2.5.2. Didactic analysis

In a first didactic step (1), it is realized that the derived density $\rho_{\Lambda,\text{theo}}$ is constant, as there is no change of the physical conditions in the process of GFV by LFV in a universe consisting of volume only. This step has no special learning barrier.

In a second didactic step (2), it is realized that the derived density $\rho_{\Lambda,\text{theo}}$ describes the density of volume, as there is no other component in a universe consisting of volume only. This step has no special learning barrier.

In a third didactic step (3), it is realized that the derived density $\rho_{\Lambda,\text{theo}}$ is the same for all times t_1 in which the process of GFV by LFV is analysed. This step has no special learning barrier, as Eq. {26} is derived in a straight forward manner. Of course, nobody can directly measure the rate H_1 at another time t_1 after the Big Bang.

2.5.3. Experience: learning process and learners

The experiences with learning groups have been documented in terms of photographs of the blackboard and with help of additional reports. These are summarized as follows.

Since 2021, see Carmesin (2021a), the topic has been presented in five general studies courses at the university. The learning process was enriched by a permanent discussion of the achieved results and by exercises about the derived relations. In particular, the learning process took place as follows:

The law of locally formed volume has been treated in advance. The learning process of the process of GFV by LFV takes 90 minutes. This includes the derivation of $\rho_{\Lambda,\text{theo}}$ and Eqs. {21} to {25}, the exercises and discussion.

A quantum gravity group of a research club meets 90 minutes each week. Thereby topics such as quantum computers, cosmology, astrophysics or quantum gravity are treated. In that group, essentially the same learning process has been treated in four courses since 2021. Also in this case, all questions have been discussed directly, and exercises have been performed.

Altogether, in all nine learning groups, the learners asked questions. These have been discussed directly in a fully sufficient manner. Moreover, exercises have been used in order to achieve sufficient training, metacognitive activity and familiarity with the new concepts. In some of the exercises, the students were instructed so that they were able to achieve parts of the derivations on their own. This is an efficient test of the ability of the students, and it provides self-esteem to the students in a convincing manner.

3. Discussion

Space is an ubiquitous entity of everyday life. Moreover, it is essential in physics, see e. g. Newton (1668) or Maxwell (1865). Einstein (1917) proposed that space has a cosmological constant Λ. It corresponds to an energy density u_Λ of space. The corresponding energy is called dark energy, see e. g. Workman et al. (2022). Observation shows that the dark energy amounts to more than 67 % of the whole energy of the universe, including matter, see Planck collaboration (2020). Thus, the question arises what properties this enormous amount of energy has.

For it, we analyse the process of global formation of volume, GFV, by local formation of volume, LFV. In particular, we analyse the ideal case of a universe consisting of volume only. As a result, we derive a formula $u_{\Lambda,\text{theo}} = \frac{c^2 H_0^2}{4\pi G}$. It is in precise accordance with observation. This fact provides convincing additional evidence for the used law of local formation of volume, which is derived from evident properties of volume, see Carmesin (2024a, f).

Moreover, the formation of volume in a universe with volume, radiation and matter has been investigated additionally, and it provides the solution of the Hubble tension, see Carmesin (2021a-b, 2023a, e, 2024d).

Altogether, we show how the volume of the universe forms in a permanent process. That process is an example of the more general dynamics of volume in nature, the volume dynamics, VD. That VD provide quantum physics and general relativity as special cases and solve many problems of physics beyond quantum physics and general relativity, see Carmesin (2021a, 2023a, e, 2024a-g).

The learning process is based on the hypothetic deductive method, see the section about the epistemology. Such a testing of a hypothesis and such a deduction from prior knowledge have a high learning efficiency, see Hattie (2006). Moreover, the learning process uses everyday life contexts, so that the learning is meaningful, see Muckenfuß (1995) and achieves an additional high learning efficiency, see Hattie (2006). In the particular case, applications to quantum cryptography and quantum computing are very motivating. For more interesting examples, see Carmesin (2020c).

The learning process of the process of GFV by LFV has been tested in nine learning groups. The learning process takes 90 minutes at university courses as well as in research club courses. In all these learning groups, the students were able to perform exercises and to use instructions in order to derive parts of the theory. Thus, the topic provides a large amount of self-esteem to the learners.

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Acknowledgement

I am very grateful to I. Carmesin for many interesting discussions and for proofreading the manuscript. I am also grateful to Matthias Carmesin for interesting discussions.