Students Learn to Derive Universal Properties of Gravitons

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Abstract

The gravitational force is a fact of everyday life. But how does it propagate from a field generating mass M to a probe mass m? For instance, on its way from Earth to Moon, Apollo 11 had to overcome the gravitational force that Earth exerts upon Apollo 11. How does this force come from Earth to Apollo 11? Newton (1668) realized that that gravitational force is fundamental, without explaining its mechanism. Einstein (1915) proposed that a field generating mass M curves space in its vicinity, so that a probe mass m moves according to that curved space. But how does the curvature propagate? For it Blokhintsev and Galperin (1934) proposed a graviton. Carmesin (2023a) derived the dynamics of volume in nature, the volume dynamics, VD. It bridges general relativity, GR, and quantum physics, QP. This additionally serves the ambitious aim to bridge gaps or differences, see Niedersächsisches Kultusministerium (2021). In this paper, we use the VD to derive insightful and useful universal properties of the proposed graviton. For that derivation, we provide a learning process, so that you can directly use the concept in your courses. The learning process has been tested in various learning groups, and experiences are reported.

1. Introduction

1.1. On the propagation of gravity

On its way from Earth to Moon, Apollo 11 had to overcome the gravitational force that Earth exerts upon Apollo 11, Fig. (1). This is an example, in which gravity propagated from Earth to Apollo 11. In the standard model of elementary particles, that propagation of gravity should be instantiated by an elementary particle, the graviton, see Workman et al. (2022).

Carmesin (2023a, 2024a) derived the VD by using evident and reliable properties of volume in nature. This is insight- & useful. Here, the epistemology is based on the established hypothetic deductive method. It is described in the didactic literature, see e. g. Kircher, Girwidz und Häußler (2001, section 4.1.2) and in the epistemological literature, see e. g. Niiniluoto, Sintonen, Wolenski (2004, S. 214). This method is used here as well as in Carmesin (2024a-g, 2017, 2018a-b, 2019a-b, 2020a-c, 2021a-d, 2022a-c, 2023a-f).

Derived universal properties include the wave property, the tensor property, the minimal dimension of space, the zero-point energy, the energy spectrum and the spin. These properties correspond to the graviton proposed by Blokhintsev and Galperin (1934).

Derived results have been used to solve problems of fundamental physics in gravity, GR, & QP.

1.2. On the dynamics of volume in nature

In this section, we summarize results of the VD, see e. g. Carmesin (2021a, 2023a, e, 2024a-g).

1.2.1. Existence of volume-portions

The evident fact that volume has no rest mass, $m_{vol,0} = 0$, implies that the volume in nature propagates at c. Moreover, volume consists of many volume-portions, VPs. A proof is presented in a parallel paper in this report about the DPG conference 2024 in Greifswald, see Carmesin (2024a).

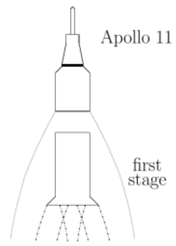

Fig. 1: Apollo 11 on its way to the Moon: The separation of the first stage is observed with a telescope. In this perspective, the length appears reduced.

1.2.2. Measurable additional volume

In the vicinity of a mass M or an effective mass M_{eff}, the light-travel distance d_{LT} can be measured, for instance with light signals, see Fig. (2). Moreover, at the same place, the gravitational parallax distance d_{GP} can be measured. It describes lengths for the case of zero mass or effective mass. The d_{GP} can be measured with help of a pair of hand leads. Proof: Carmesin (2023a, 2024a). An increment of volume dV_L of a cuboid is the product of d_{LT} based lengths of the edges. Similarly, the product of the corresponding

d_{GP} based lengths is the volume dV_R of the cuboid that would occur at zero M or M_{eff}. The difference $\delta V = dV_L - dV_R$ is the additional volume caused by M or M_{eff}. The relative additional volume is $\varepsilon_L = \frac{\delta V}{dV_L}$, see Carmesin (2023a, 2024a).

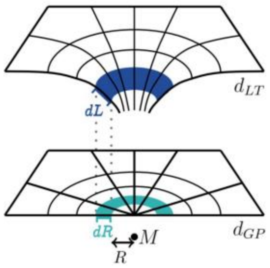

Fig. 2: In the vicinity of M or M_{eff}, the radial increment dL of d_{LT} is increased with respect to the original increment dR that would occur in the limit M to zero.

1.2.3. Propagation of volume

(1) Localizable portions of relative additional volume ε_L and of volume in general propagate, see Fig. (3). This is described by the law of propagation of localizable relative additional volume. E. g., the parts (2) and (3) in that law state the following:

(2) If the relative additional volume ε_L is analysed as a function of τ and $\vec{L}$, see Fig. (3), then it fulfils the following differential equation, DEQ:

$$\frac{\partial}{\partial \tau}\, \varepsilon_L = -c \cdot \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}}\, \varepsilon_L \qquad \{1\}$$

Hereby, $\vec{e}_v$ is the radial direction vector, it is also marked by $\vec{e}_L$.That Eq. implies the Schrödinger equation, SEQ. Proof: Carmesin (2024a).

(3) In principle, there is no difference between a portion δV and a localizable VP. Thus, Eq. {1} holds for each localizable VP. A proof is in Carmesin (2024a).

1.2.4. Curvature caused by volume-portions

In the vicinity of M or M_{eff}, space is curved, Fig. (2). This is implied by Eq. {1}. Proof: Carmesin (2024a).

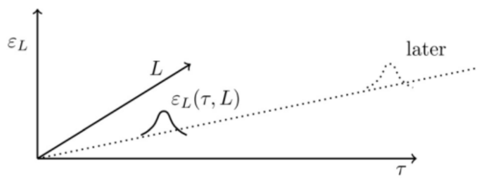

Fig. 3: A localizable portion of relative additional volume ε_L propagates in space. The relative additional volume is analysed as a function of τ and $\vec{L}$.

1.2.5. Gravitational potential and field

The dynamics in Eq. {1} can be written as follows:

$$c\,\frac{\partial}{\partial \tau}\, \varepsilon_L = \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} \cdot (-c^2 \cdot \varepsilon_L) = \vec{e}_v \cdot \frac{\partial}{\partial \vec{L}} \Phi_{gen} \quad \{2\}$$

The bracket in the above DEQ has the form of a generalized potential Φ_{gen}: $\Phi_{gen} := -c^2 \cdot \varepsilon_L \quad \{3\}$

The negative gradient of that generalized potential is the generalized field $\vec{G}_{gen}$:

$$\vec{G}_{gen} := -\frac{\partial}{\partial \vec{L}}\,(-c^2 \cdot \varepsilon_L) = -\frac{\partial}{\partial \vec{L}}\,\Phi_{gen} \qquad \{4\}$$

These results are useful, as they provide the curvature in section (1.2.4) and as the field $\vec{\boldsymbol{G}}_{\boldsymbol{gen}}$ in Eq. {4} describes gravity exactly. Proof: Carmesin (2024a).

2. Didactic analysis

2.1. Rate gravity waves

DEQ {1} has harmonic solutions:

2.1.1. Physical analysis

Harmonic waves are solutions of DEQ {2}:

$$\varepsilon_L = \hat{\varepsilon}_{L,\omega} \cdot \exp\left(-i\omega t + i\,\vec{k} \cdot \vec{L}\right) \qquad \{5\}$$

$$\Phi_{gen} = \widehat{\Phi}_{gen,\omega} \cdot \exp\left(-i\omega t + i\,\vec{k} \cdot \vec{L}\right) \qquad \{6\}$$

Inserting in Eq. {2} yields: $-i\omega c \hat{\varepsilon}_{L,\omega} \cdot \exp\left(-i\omega t + i\,\vec{k} \cdot \vec{L}\right) = i\vec{k}\,\vec{e}_L\,\widehat{\Phi}_{gen,\omega} \cdot \exp\left(-i\omega t + i\,\vec{k} \cdot \vec{L}\right)$

In the above Eq., the relations $\vec{k}\,\vec{e}_L = k$ and $c = \frac{\omega}{k}$ are used. Thus: $-\,c^2 \hat{\varepsilon}_{L,\omega} = \widehat{\Phi}_{gen,\omega} \qquad \{7\}$

As a consequence, the solution in Eq. {6} can be expressed as follows: $\Phi_{gen} = -c^2 \cdot \varepsilon_L \qquad \{8\}$

This relation confirms the potential in Eq. {3}.

In order to provide a more physical interpretation, the derivatives in the DEQ {2} are evaluated as follows:

$$c\,\dot{\varepsilon}_L = -\,\vec{e}_v \cdot \vec{G}_{gen} \qquad \{9\}$$

In the above Eq., $\dot{\varepsilon}_L$ describes a 'rate of the change' of relative additional volume, and $\vec{G}_{gen}$ describes gravity. Accordingly, the solution in Eqs. {5} and {6} is called rate gravity wave, RGW, Carmesin (2021a).

Based on polar representation of complex numbers,

$$\exp(\mathrm{i}\alpha) = \cos(\alpha) + \mathrm{i} \cdot \sin(\alpha), \qquad \{10\}$$

the waves in Eqs. {5} and {6} can be expressed in terms of sine and cosine functions, if desired.

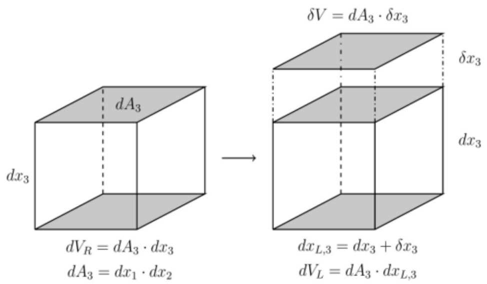

Fig. 4: VPs can be represented with cubes and cuboids.

2.1.2. Didactic analysis

In a first didactic step, a harmonic wave in Eqs. {5} and {6} is used as a solution of DEQ {2} of VD. It can be visualized with help of sine and cosine functions. This step has no special learning barrier.

In a 2^{nd} didactic step, a harmonic wave in Eqs. {5} & {6} is verified. Moreover, it is interpreted as a rate gravity wave. The step has no special learning barrier.

Both steps are very insightful and valuable as the rate gravity waves indicate how the exact gravitational field $\vec{G}_{gen}$ propagates in combination with the 'rate of change' $\dot{\varepsilon}_L$ of relative additional volume.

2.2. Tensor property of volume in nature

2.2.1. Physical analysis

Theorem: Law of measurable changes of VPs: In the vicinity of each M or M_{eff}, the following changes of VPs can be measured: (1) additional volume, (2) shear, (3) rotation, (4) translation and (5) linear combinations thereof:

(1) The diagonal tensor elements of change represent the change of a volumetric property, Fig. (4):

$$\varepsilon_{L,jj} = \frac{\delta x_j}{dx_{L,j}} = \frac{\sqrt{|g_{jj}|}-1}{\sqrt{|g_{jj}|}} = 1 - \frac{1}{\sqrt{|g_{jj}|}} \quad \{11\}$$

$\varepsilon_{L,jj}$ is a unidirectional change in direction j.

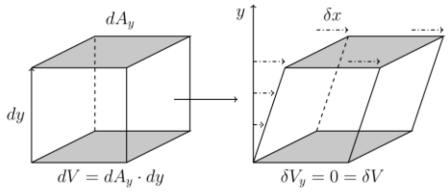

Fig. 5: The left VP is a cube, and it is changed by the operation of shear.

(2) The non-diagonal symmetric tensor elements of change represent shear:

$$\varepsilon_{L,ij} = \frac{1}{2}\left(\frac{\delta x_i}{dx_{L,j}} + \frac{\delta x_j}{dx_{L,i}}\right) \quad for \;\; i \neq j \quad \{12\}$$

These tensor elements constitute the non-diagonal symmetric tensor of change, see Fig. (5).

(3) The non-diagonal antisymmetric tensor elements of change represent rotation:

$$\varepsilon_{L,ij} = \frac{1}{2}\left(\frac{\delta x_i}{dx_{L,j}} - \frac{\delta x_j}{dx_{L,i}}\right) \quad for \;\; i \neq j \quad \{13\}$$

These tensor elements constitute the non-diagonal antisymmetric tensor of change.

(4) A translation of a VP at a position $\vec{L}$ is represented as follows, see Fig. (3): $\delta\vec{L} = \frac{\partial\vec{L}}{\partial\tau} \cdot \delta\tau \quad \{14\}$

The corresponding coordinates of translation are as follows: $\delta L_j = \frac{\partial L_j}{\partial\tau} \cdot \delta\tau \quad \{15\}$

Changes of relative additional volume are described by its law of propagation.

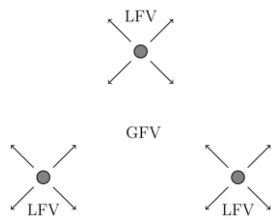

Fig. 6: Unidirectional **l**ocal **f**ormation of **v**olume, LFV, at masses or dynamic masses can summarize to isotropic **g**lobal **f**ormation of **v**olume, GFV.

(5) Linear combinations of the changes in (1) to (4) can be measured. In particular, the isotropic change can be measured, see Fig. (6):

$$\varepsilon_{L,iso} = \sum_j^D \varepsilon_{L,jj} \quad with \;\; \varepsilon_{L,ii} = \varepsilon_{L,jj} \quad \{16\}$$

Proof: Part (1): d_{GP} distances dx_j and d_{LT} distances $dx_{L,j}$ can be measured. Thus, changes of VPs in parts (1) to (5) can be measured. Q. e. d.

2.2.2. Didactic analysis

In step (1), unidirectional change is analysed. This step has no special learning barrier. It is insightful that additional volume is not necessarily isotropic.

In step (2), shear is analysed. This step has no special learning barrier. It is insightful that volume can change without forming any additional volume.

In step (3), rotation of a VP is analysed. This step has no special learning barrier. Note that VPs can rotate.

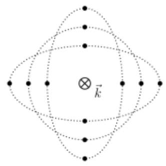

Fig. 7: A gravitational wave propagates in a direction $\vec{k}$. In the plane orthogonal to $\vec{k}$, the wave exhibits positive and negative additional volume in a periodic manner.

In step (4), the translation is analysed. It has no special learning barrier. It is insightful to realize that the propagation corresponds to the isometry of translation. In principle, a reflection could also be possible.

In step (5), linear combinations are analysed. In particular, isotropic formation of additional volume is presented. It has been essential in the isotropic expansion of space since the Big Bang. It represents global formation of volume, GFV, and it can be derived from local formation of volume, LFV, see e. g. Carmesin (2023a). This step has no special learning barrier.

2.3. Dimension of volume in nature

In order to analyse possible dimensions of volume or space in nature, we study gravitational waves:

2.3.1. Physical analysis

(1) A gravitational wave propagates in a direction $\vec{k}$, and it exhibits changes in the direction orthogonal to $\vec{k}$. These can be described by the following tensor of relative additional volume in the plane orthogonal to $\vec{k}$, and there are two linear polarisations, see e. g. Abbot et al. (2016) or Carmesin (2017b) and Fig. (7).:

$$\varepsilon_{ij} = \hat{\varepsilon} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \& \quad \varepsilon_{ij} = \hat{\varepsilon} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; \{17\}$$

As a consequence, gravitational waves require a space that has at least three dimensions. Moreover, gravitational waves are waves of periodic variation of additional volume in nature, see Eq. {17}.

(2) Binary stars can lose kinetic energy with help of gravitational waves in an efficient manner. Consequently, gravitational waves are important for mergers of binary stars or black holes. Such mergers are essential for the formation of structure in the universe.

(3) An additional information may be considered: The time evolution of space since the Big Bang has been analysed with help of additional volume that propagates in the form of gravitational waves. As a test, the energy density ρ_Λ of the cosmological constant Λ has been derived. It is in precise accordance with observation, see Carmesin (2018a-b, 2019a-b, 2021a-d). Thus, space needs to have at least three dimensions.

(4) In physics, subsystems of three-dimensional space are often analysed. For instance, graphene is often modelled as a two-dimensional system. The investigation of such subsystems does not change the fact that present – day space has at least three dimensions.

2.3.2. **Didactic analysis**

In a first step, the physics of gravitational waves are summarized. This step has no special learning barrier.

In a second didactic step, binary stars are analysed. This step has no special learning barrier.

Step (3) provides information. It is no didactic step.

Step (4) provides a clarification of three-dimensional space and the analysis of a lower-dimensional subsystem. This step has no special learning barrier.

2.4. **Gaussian wave packets**

2.4.1. **Physical analysis**

(1) The following Gaussian wave packets are solutions of DEQ {1} for $\varepsilon_L(\tau, \vec{L})$. The time derivative of that DEQ provides the following DEQ for $\partial_\tau \varepsilon_L = \dot{\varepsilon}_L$:

$$\frac{\partial}{\partial\tau}\, \dot{\varepsilon}_L = -c \cdot \vec{e}_v \cdot \frac{\partial}{\partial\vec{L}}\, \dot{\varepsilon}_L \qquad \{18\}$$

This is the generalized Schrödinger equation, whereby $\Psi = t_n \cdot \dot{\varepsilon}_L$ is the wave function and t_n is a normalization factor, see e. g. Carmesin (2024a, 2023a, 2022a-b). Thus, the above DEQ holds for Ψ as well. As the above DEQ for Ψ is the same as DEQ {1} for ε_L, the analysis of the respective solutions is equivalent.

In a one-dimensional subspace with a coordinate x_j, a Gaussian wave packet with a characteristic wave vector $k_{0,j}$, can be described as follows, see Landau and Lifschitz (1965) or Greiner (1979, chapter 3):

$$\Psi_{\mathrm{j}} = \nu_j \cdot \exp\left(i \cdot k_{0,j} \cdot x_j - \frac{x_j^2}{4\sigma_j^2}\right)$$

Hereby, $\nu_j = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{\sigma_j}}$ is the normalization factor.

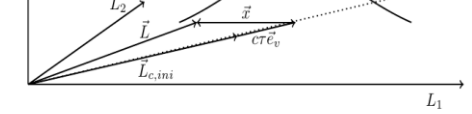

Fig. 8: VP with an initial position of the centre at $\vec{L}_{c,ini}$. The VP moves, during a time τ, the centre moves to $\vec{L}_{c,ini} + c\tau\, \vec{e}_v$. The VP has a form (similar to an orbital in an atom). A part is shifted by a vector $\vec{x}$ from the centre, so the part is at a coordinate $\vec{L}$, in an external frame.

In an external frame, see Fig. (8), the object propagates at the velocity c. The corresponding wave function is achieved by substituting x_j by $L_j - L_{c,j}$. Hereby, $L_{c,j}$ is the coordinate of the local maximum of Ψ. This is similar to the local maximum of ε_L in Fig. (3). Moreover, the local maximum propagates from an initial value $L_{ini,j}$ to the following values:

$$L_{c,j} = L_{c,ini,j} + c \cdot \tau \;\&\; x_j = L_j - L_{c,ini,j} - c \cdot \tau$$

Consequently, the wave function is as follows:

$$\Psi_{\mathrm{j}} = \nu_j \cdot \exp\left(ik_{0,j}(L_j - L_{c,j}) - \frac{\left(L_j - L_{c,j}\right)^2}{4\sigma_j^2}\right)$$

We check that the wave function solves DEQ {18}:

$$i\hbar\partial_\tau\Psi_{\mathrm{j}} = -i\hbar c\partial_{L_j}\Psi_j \;, \qquad \text{with}$$

$$\partial_{L_j}\Psi_j = ik_{0,j}\Psi_j - \frac{L_j - L_{c,j}}{2\sigma_j^2}\Psi_j \quad \text{and}$$

$$\partial_\tau\Psi_j = -ik_{0,j}c\Psi_j + c\,\frac{L_j - L_{c,j}}{2\sigma_j^2}\Psi_j, \text{thus}$$

$$\hbar ck_{0,j}\Psi_{\mathrm{j}} + i\hbar c\,\frac{L_j - L_{c,j}}{2\sigma_j^2}\Psi_{\mathrm{j}}$$
$$= \hbar ck_{0,j}\Psi_j + i\hbar c\,\frac{L_j - L_{c,j}}{2\sigma_j^2}\Psi_{\mathrm{j}}$$

This shows that the above Gaussian wave packet obeys the DEQ {18}. In contrast, in the non-relativistic SEQ, a Gaussian wave packet broadens in its time evolution. It is a consequence of $\omega \propto k^2$ in the non-relativistic SEQ, see Greiner (1979, chapter 3).

In fact, it is a very valuable property of the DEQ {18} and of the volume in nature, that Gaussian wave packets are stable. The reason for it is that $m_{0,vol} = 0$.

In 3D space, the wave function is, see Fig. (8):

$$\Psi = \nu \cdot \exp\left(i\,\vec{k}_0\vec{x} - \frac{\vec{x}^2}{4\sigma_j^2}\right) \;\&\; \vec{x} = \vec{L} - \vec{L}_{c,ini} - c\tau\vec{e}_v$$

This Ψ solves DEQ {1}, see Carmesin (2024g), and Ψ does not broaden during the time evolution.

2.4.2. **Didactic analysis**

In step(1), a Gaussian wave packet of a VP is presented in a one-dimensional subsystem. For it, an external frame is introduced. Moreover, the wave function is verified as a solution of the DEQ {18}. This step has no special learning barrier. The step is very valuable, as the wave packets do not broaden as a function of time. This is an important insight, as it shows that wave functions that do not broaden are fundamental, they have $m_{0,vol} = 0$, and they solve the fundamental VD in DEQ {18}. In contrast, the broadening of wave functions is a particular phenomenon that occurs in wave packets of masses, since masses change the relation between ω and k, as a consequence of the energy mass relation $E^2 = p^2c^2 + m_0^2c^4$.

In step (2), the Gaussian wave packet in a one-dimensional subsystem is transferred to three-dimensional space. This step has no special learning barrier. The step is valuable, as Gaussian wave packets do not broaden in 3D space.

2.5. Minimal energy of volume-portions

2.5.1. Physical analysis

(1) The SEQ exhibits the Heisenberg uncertainty relation, Carmesin (2024g): $\Delta x_j \cdot \Delta p_j \geq \frac{\hbar}{2}$ {19}

Gaussian wave packets exhibit, Carmesin (2024g):

$\Delta x_j \cdot \Delta p_j = \frac{\hbar}{2}$, in Gaussian wave packets {20}

In D dimensional space, see Olofsson and Andersson (2012, proposition 2.5.1): $\Delta|\vec{x}| = \Delta x_j \cdot \sqrt{D}$ {21}

Similarly: $\Delta|\vec{p}| = \Delta p_j \cdot \sqrt{D}$ {22}

Thus: $\Delta|\vec{x}| \cdot \Delta|\vec{p}| = \frac{D \cdot \hbar}{2}$ {23}

(2) As a consequence, the fluctuations of the vector occur at a ball with the radius $\Delta|\vec{x}|$. The fluctuation with the minimal energy E_{min} is the fluctuation with the largest wavelength: $\lambda_{max} = 2\pi \cdot \Delta|\vec{x}|$ {24}

The circular frequency is: $\omega = \frac{2\pi c}{\lambda_{max}}$ {25}

Eqs. {24-25} yield: $\omega = \frac{c}{\Delta|\vec{x}|}$ {26}

The minimal energy E_{min} is equal to $c \cdot \Delta|\vec{p}|$. Thus, Eqs. {23} & {26} yield: $\mathrm{E_{min}} = \frac{D \cdot \hbar \cdot c}{2\Delta|\vec{x}|}$ {27}

$\mathrm{E_{min}} = \mathrm{D} \cdot \frac{\hbar \cdot \omega}{2}$, in $D -$ dimensional space {28}

(3) If a one-dimensional subsystem is analysed, then the minimal energy of a VP is equal to the above energy divided by D:

$\mathrm{E_{min}} = \frac{\hbar \cdot \omega}{2}$, in a 1-dimensional subsystem {29}

(4) The above energy of the fluctuations includes no potential or field. Consequently, it is a completely kinetic energy. This fact is also confirmed in parallel papers in this report about the DPG conference 2024 in Greifswald.

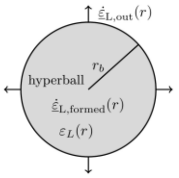

Fig. 9: A localizable and stationary portion of relative additional volume ε_L in its own system.

2.5.2. Didactic analysis

In step (1), the uncertainty relation is applied to a VP in its own frame and in an external frame in various dimensions. This step has no special learning barrier. The step is insightful, as it shows how fluctuations combine in several dimensions.

In a second didactic step, the VP with minimal energy is analysed. This step has no special learning barrier. The step is insightful and very valuable, as it provides the energy of the minimal uncertainty at the VP.

In a third didactic step, the result is transferred to a one-dimensional subsystem. This step has no special learning barrier. The step is useful, as the complexity of a three-dimensional system can often be reduced with help of one-dimensional subsystems.

In step (4), fluctuation energy is identified with kinetic energy. For it, other forms of energy have been excluded. This step has no special learning barrier.

2.6. Wave packet with minimal energy

In this section, a derived, modified, corrected, generalized version of quantum field theory is presented. The proofs are in Carmesin (2024g).

2.6.1. Physical analysis

Theorem: Transformation of a wave packet:

(1) A Gaussian wave packet $\varepsilon_{L,p}(\tau, \vec{L})$, with a polarization p can be transformed with Fourier integrals, see e. g. Landau and Lifschitz (1965, § 5 and § 15):

$b_\mu(\tau) = \nu_b \cdot \exp(-i\omega_\mu \tau)$ {30}

$\langle b_\mu | b_{\mu'} \rangle_t = \int_{-\infty}^{\infty} b_\mu(\tau) b_{\mu'}^{cc}(\tau) d\tau = \delta(\mu - \mu')$ {31}

Hereby, cc marks the conjugate complex. In general, the index μ includes possible polarization directions or indices marking tensors. Moreover, $\langle b_\mu | b_{\mu'} \rangle$ is a scalar product in the Hilbert space of the solutions of the DEQ {1}, and $\delta(\mu - \mu')$ is the Dirac delta function (or delta distribution), see Kumar (2018). ν_b, ν_f are normalisation factors.

$f_\mu(\vec{k}_\mu, \vec{L}) = \nu_f \cdot \exp(i\, \vec{k}_\mu \cdot \vec{L})$ {32}

$\langle f_\mu | f_{\mu'} \rangle = \int\int \int_{-\infty}^{\infty} f_\mu f_{\mu'}^{cc} d^3L \propto \delta(\vec{k}_\mu - \vec{k}_{\mu'})$ {33}

$\langle f_\mu | f_{\mu'} \rangle = 4\pi \int_0^{\infty} f_\mu(k_\mu, L) f_{\mu'}^{cc}(k_{\mu'}, L) L^2 dL$ {34}

$\langle f_\mu | f_{\mu'} \rangle = 4\pi \cdot \delta(k_\mu - k_{\mu'})$ {35}

(2) The transformed wave packet is described by $\varepsilon_L(\tau, \vec{L})$, a potential $\Phi_{gen}(\tau, \vec{L})$, and amplitudes $\hat{\varepsilon}_\mu$:

$\varepsilon_L(\tau, \vec{L}) = \int d\mu\, \hat{\varepsilon}_\mu \cdot b_\mu(\tau) \cdot f_\mu(\vec{k}_\mu, \vec{L})$, {36}

$\Phi_{gen}(\tau, \vec{L}) = \int d\mu\, \widehat{\Phi}_{gen,\mu} \cdot b_\mu(\tau) \cdot f_\mu(\vec{k}_\mu, \vec{L})$, {37}

The transformed energy is:

$E_\mu = \frac{c^2}{2G} \hat{\varepsilon}_\mu \hat{\varepsilon}_\mu^{cc} b_\mu b_\mu^{cc} (\omega_\mu^2 - c^2 k_\mu^2)$ & $E = \int d\mu E_\mu$ {38}

Theorem: Eigenvalue generating operator:

(3) E_μ is expressed with: $q_\mu = \frac{c}{\sqrt{G}} \cdot \hat{\varepsilon}_\mu b_\mu$ {39}

$p_\mu := \frac{\partial}{\partial \tau} q_\mu = i \cdot \omega_\mu q_\mu = \frac{i c^2 \hat{\varepsilon}_\mu b_\mu k_\mu}{\sqrt{G}}$ {40}

$E_\mu = \frac{1}{2}(q_\mu q_\mu^{cc} \omega_\mu^2 - p_\mu p_\mu^{cc})$ {41}

The derivative $\frac{\partial}{\partial \tau}$ is irreversible. The generalized kinetic energy is: $E_{gen,kin,\mu} = \frac{q_\mu q_\mu^{cc} \omega_\mu^2}{2}$ {42}

The squared wave vector is: $k_\mu^2 = \sum_j^3 k_{\mu,j}^2$ {43}

(4) Eigenvalue generating operators are introduced:

$\hat{k}_\mu = \frac{i}{c} \cdot \frac{\partial}{\partial \tau}$ or $\hat{p}_\mu = i \cdot \frac{\partial}{\partial \tau} q_\mu$, thus {44}

$\hat{E}_\mu = \frac{1}{2}\left(q_\mu q_\mu^{cc}\omega_\mu^2 - \hat{p}_\mu \hat{p}_\mu^{cc}\right)$ {45}

The time averaged commutator is:

$\left\langle \left[q_\mu , p_{\mu'}^{cc}\right] \right\rangle_t = i\omega_\mu \cdot q_\mu q_\mu^{cc} \cdot \delta(\mu' - \mu)$ {46}

Theorem: Excitation generating operator:

(5)Ladder operators: $q_\mu = \frac{\alpha}{\omega_\mu} \cdot \left(a_\mu^+ + a_\mu\right)$ {47}

$p_\mu = i \cdot \alpha \cdot \left(a_\mu^+ - a_\mu\right)$ {48}

α is chosen so that:$\left[a_\mu , a_{\mu'}^+\right] = \delta(\mu' - \mu)$ {49}

Thus: $\alpha^2 = \frac{\omega_\mu^2 q_\mu q_\mu^{cc}}{2}$ {50}

$\hat{E}_{\mu,1D} = 2\,\alpha^2 \cdot \left(a_\mu^+ a_\mu + \frac{1}{2}\right)$ {51}

Theorem: number- and energy spectrum:

(6) Eigenvalues n_μ and eigenvectors $|n_\mu\rangle$ of the number operator in $\hat{E}_{\mu,1D}$: $N_\mu := a_\mu^+ a_\mu$ {52}

$N_\mu \left|n_\mu\right\rangle = n_\mu |n_\mu\rangle;\; a_\mu \left|n_\mu\right\rangle = \sqrt{n_\mu}\left|n_\mu - 1\right\rangle$ {53}

$a_\mu^+ \left|n_\mu\right\rangle = \sqrt{n_\mu + 1}\left|n_\mu + 1\right\rangle$ {54}

$\left\langle n'_\mu \middle| a_\mu \middle| n_\mu \right\rangle = \sqrt{n_\mu}\delta_{n'_\mu, n_\mu - 1}\;\; for\;\; n_\mu > \;\; 0$ {55}

a_μ^+: raising operator, a_μ: lowering operator with, $a_\mu|0\rangle = 0;\;\; a_\mu|1\rangle = |0\rangle$. The number spectrum is as follows: $n_\mu \in \{\,0, 1, 2, 3, \ldots\}$ {56}

Thus, the lowest energy is the zero-point energy, ZPE: $E_{\mu,1D} = \frac{1}{2} \cdot \hbar\omega_\mu = ZPE_\mu$ {57}

(7) Interpretation: Firstly, the ZPE_μ in Eq. {57} is equal to the kinetic energy.

Secondly, we derived that the complete energy of the analysed VP is zero, see Eq. {38} and $c = \frac{\omega_\mu}{k_\mu}$.

Thirdly, the question of the origin of the nonzero ZPE_μ arises. This origin is traceable: That ZPE_μ originates from the commutator. That nonzero commutator arises from the derivative. That derivative is not reversible. Thus, the theory based on that derivative can only predict differences and correlations, but not absolute values. These absolute values are predicted by the present theory of VD: The VD provides the complete ZPE, it is zero:

$E_\mu = \frac{c^2}{2G}\hat{\varepsilon}_\mu \hat{\varepsilon}_\mu^{cc} b_\mu b_\mu^{cc}\left(\omega_\mu^2 - c^2 k_\mu^2\right) = 0$ {58}

Hereby, we used $c = \frac{\omega_\mu}{k_\mu}$. Moreover, the above Eq. shows that the potential energy compensates the generalized kinetic energy of the ZPE. In contrast, the ZPE in $E_{\mu,1D}$ provides the information about the generalized kinetic energy only, so it is incomplete.

The full energy spectrum is obtained from the ZPE state $|0\rangle$ by application of the ladder operator a_μ^+.

The incomplete ZPE in 3D space consists of three one-dimensional projections of the ZPE:

$E_{\mu,3D} = \frac{3}{2} \cdot \hbar\omega_\mu = ZPE_{\mu,3D}$ {59}

2.6.2. Didactic analysis

In step (1), we summarize the Fourier transformation. It is explained with the analogues of optical and acoustic spectral analysis. Thus, it can be understood in principle, moreover, the algebra can be confirmed.

This step has a medium sized learning barrier, as two frames are considered in the Gaussian wave packet: In an external frame, the Gaussian wave packets propagate. In the own frame, there is a Gaussian function.

In step (2), the transformed energy is derived algebraically. So, there is no special learning barrier.

In step (3), abbreviations q_μ and p_μ are introduced. This step has no special learning barrier.

In step (4), eigenvalue generating operators are introduced and used. Algebraically, this step has no special learning barrier. Moreover, it is clear that the derivative cannot be inverted. This provides a medium-sized mental barrier. This barrier is overcome by a discussion of the consequences.

In step (5), ladder operators are introduced by a linear transformation. There is no special learning barrier.

In step (6), the number operators and their spectrum are derived. This step has no special learning barrier.

Moreover, the energy is interpreted. This step has a medium-sized mental learning barrier, as it must be realized, that the derived algebra provides differences only. Thus, the absolute value of the energy is derived with help of the VD. Altogether, the algebra of QFT is derived. In contrast, before, QFT has been regarded as a set of ideas and tools, see Peskin and Schroeder (1995). As a consequence, the present derived version has the correct and non-diverging energy, in contrast to diverging energy in present – day QFT. Thus, the present derived version of QFT is modified and corrected and generalized.

2.7. Spin

2.7.1. Physical analysis

(1) The spins and tensors can be characterized by their behaviour with respect to a rotation in three-dimensional space, see Landau and Lifschitz (1965, § 58).

(2) Rank two tensors describe the VPs. Moreover, rank two tensors have periodicity $\varphi_{per} = \pi$, see Carmesin (2024g).

(3) According to the VD, wave functions are described by the Schrödinger Eq., SEQ, or by the generalized Schrödinger Eq., GSEQ, see Carmesin (2024a,g). In a magnetic field $\vec{B}$, the energy of a

particle with spin $\vec{S}$ is as follows, see Sakurai (1994, section 3.2, we use SI units):

$$H = -\frac{e}{m_e}\vec{B} \cdot \vec{S} = \omega S_z;\ \omega = \frac{|e|B}{m_e};\ S_z = \frac{\hbar}{2} \quad \{60\}$$

Hereby, m_e is the mass of the electron, S_z is the eigenvalue of the spin in the z-direction, and ω is the circular frequency of the precession. As a consequence of the SEQ or GSEQ, the periodicity is, see e. g. Carmesin (2024g): $\varphi_{per} = \frac{2\pi\hbar}{S_z}$ {61}

In particular, $\varphi_{\mathrm{per}} = \pi$ for $\mathrm{S} = 2\hbar$ {62}

(4) As a consequence of items (1-3), VPs have spin 2.

This result is in accordance with the usually assumed value, see Workman et al. (2022).

2.7.2. Didactic analysis

In a first didactic step, the criterion of equal periodicity is introduced. This provides no special difficulty.

In step two, the periodicity of tensors is derived. This is achieved by algebraic transformations, so it provides no special learning barrier.

In step (3), the energy & periodicity of a spin in a field are analysed. This step requires to confirm the solution of the SEQ or GSEQ, see Carmesin (2024g). Thus, the step has no special learning barrier.

In step four, the criterion in item (1) is applied to the results in (2-3). There is no special learning barrier.

The result represents an important universal property.

3. Experience: learning process and learners

Experiences with learning groups have been documented via photographs of the blackboard and via additional reports. These are summarized as follows.

VPs and DEQ {1} have been derived before.

(I) In a main block, the solutions of DEQ {1}, the transformations of these solutions, the operators q_μ, p_μ, a_μ^+, a_μ and N_μ have been derived as follows:

During the first 90 minutes, in a general studies course at the university, the harmonic solutions of DEQ {1} have been derived. For it, an Ansatz has been proposed, and the students verified that this Ansatz solves DEQ {1}. The learning process took place similarly in a research club. Moreover, these solutions have been derived in several general studies courses and research club courses since 2021. Hereby, the students achieve competence in solving DEQs.

During the second 90 minutes, in a university general studies course, the Gaussian wave packets have been derived as solutions of DEQ {1}. For it, an Ansatz has been proposed, and the students verified that the Ansatz solves DEQ {1}. The learning process took place similarly in a research club. Hereby, the students achieve competence in realizing and discussing the usefulness of wave packets in fundamental physics, see e. g. Fig. (8). Additionally, the students improve their competence in solving DEQs on their own.

During the next 45 minutes, in a general studies course, the standard deviations Δx_j, Δp_j, $\Delta|\vec{\mathrm{x}}|$ and $\Delta|\vec{\mathrm{p}}|$ have been derived. And the corresponding uncertainty relations $\Delta x_j \cdot \Delta p_j = \frac{\hbar}{2}$ and $\Delta|\vec{\mathrm{x}}| \cdot \Delta|\vec{\mathrm{p}}| = \frac{\mathrm{D}\hbar}{2}$ have been derived, for the case of Gaussian wave packets. Hereby, students achieve competence in analysing fluctuations mathematically and physically.

During the next 45 minutes, in the general studies course, the scalar products in Hilbert space and the energy E_μ have been derived. Hereby, the high activity of the students in the derivation of the solutions during the first 180 minutes of the learning process provided an effective basis for the understanding. Hereby, students achieve competence in analysing scalar products and functions in Hilbert space.

During the following 90 minutes, in the general studies course at the university, the summarizing variables q_μ and p_μ have been introduced. Moreover, the students derived the corresponding commutators.

The ladder operators have been introduced, and the students derived the respective commutators. During these derivations, the students achieve competence in analysing algebraic structures in Hilbert space.

During the following 45 minutes, in the general studies course, the number operator and the matrix elements of the ladder operators have been derived. Moreover, the spectrum has been derived. Furthermore, the students realized that the derivative causes a loss of information, so that only difference and correlations are reliable in the algebra of the ladder operators and number operators. In contrast, absolute values, such as the zero-point energy, are obtained from the original and more general VD. As the students had already experience in deriving commutators and other algebraic results from the preceding blocks, it was not necessary that the students derive all algebraic results on their own. Instead, the activity of the students was focused to the discourse and discussion. This turned out to be very appropriate, as the students achieved an overview in this manner. During these derivations, the students achieve competence in using ladder operators and number operators in Hilbert space.

(II) The remaining results have been derived:

During 90 minutes, in several general studies courses at the university since 2021, the tensor properties of the relative additional volume has been derived, see section (2.2). With it, several further results have been derived, see e. g. Carmesin (2023a). The learning process took place similarly in various research club courses since 2021. During these derivations, the students achieve competence in analysing tensors.

Since 2019, during a block of 45 minutes, and in several general studies courses at the university as well as in several research club courses, the smallest possible dimension of space has been derived, see section (2.3). During these derivations, the students achieve competence in deriving implications, and in discussing the corresponding conditions.

Since 2021, during a block of 45 minutes, and in several general studies courses at the university as well as in several research club courses, the spin of the graviton has been derived. During these derivations, the students achieve competence in deriving implications, and in discussing the corresponding conditions.

Since 2021, during a block of 30 minutes, and in several general studies courses as well as in several research club courses, the universal properties of the graviton have been discussed. During these discussions, the students achieve competence in discussing conditions, generality or universality of results.

In all these learning groups, students were able to perform exercises and to use instructions in order to derive parts of the theory. Moreover, the students discussed achieved results and analysed the corresponding conditions. Thus, the topic provides a large amount of self-esteem to the learners.

4. Discussion

The gravitational force is present in everyday life. However, the question remains: How does gravity propagate from a field generating mass to a probe mass? In present-day physics, there are two different concepts: GR proposes that curvature of space & time describe gravity, see e. g. Einstein (1915) or Hobson (2006). In contrast, in the framework of the other three fundamental interactions, a force propagates in the form of a boson of interaction, see Blokhintsev & Galperin (1934); Workman et al. (2022).

Here, the more general and fundamental dynamics of volume in nature is used: Thereby, the curvature of space is derived and treated in a parallel paper, see Carmesin (2024a) or Carmesin (2023a). In this paper, the insightful properties of the boson of interaction are derived. As a consequence, the graviton is fundamentally derived. Thus, for the first time, the graviton is fundamentally founded from a general theory, which implies gravity and GR and QP, and which solves the fundamental problems of these theories, see Carmesin (2024g). In particular, both above mentioned views are represented as special cases of the VD. In particular, the key DEQ {1} of VD is a basis.

In general, that DEQ holds for tensors, correspondingly, the tensor property of the graviton is elaborated in a clarifying manner. In fact, this is confirmed by an analysis starting at the Planck scale and providing the correct energy density of volume, see e. g. Carmesin (2018a-b, 2019a-b, 2020a-b, 2021a-b, 2023a, 2024d). With it, the insightful minimal dimension of volume in nature is derived: $D = 3$.

For the basic DEQ {1}, the valuable harmonic solutions and Gaussian wave packets are derived as solutions. Moreover, the key result of universal quantization is used in order to derive the very useful wave packets with minimal energy of the fluctuations, a kinetic energy. The result is the kinetic zero-point energy. Moreover, the insightful complete energy and the kinetic energy of a wave packet are derived.

For the case of the important kinetic energy, the minimal value as well as the excitation states, the spectrum, the ladder operators and the algebraic structure are derived. In this manner, a derived, founded valuable and enlightening modified, corrected and generalized QFT is derived.

Altogether, the analysis of the graviton provides many deep insights and useful tools, such as a founded and derived modified QFT.

The learning process is based on the hypothetic deductive method, see the section about the epistemology. Such a testing of a hypothesis and such a deduction from prior knowledge have a high learning efficiency, see Hattie (2006). Moreover, the learning process uses everyday life contexts, so that the learning is meaningful, see Muckenfuß (1995) and achieves an additional high learning efficiency, see Hattie (2006). In the particular case, applications to quantum cryptography and quantum computing are very motivating. For more examples, see Carmesin (2020c).

The learning process has been tested in several learning groups. The complete learning process takes 615 minutes at university courses as well as in research club courses. Thereby, the main part of the exact field theory, QP and derived modified QFT is most valuable and requires 360 minutes, including exercises, discussions and instructed derivations. In all these learning groups, the students were able to take part in instructed derivations and founded discussions. Thus, the topic provides a large amount of self-esteem to the learners.

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Acknowledgement

I am very grateful to I. Carmesin for many interesting discussions and for proofreading the manuscript. I am also grateful to Matthias Carmesin for interesting discussions.