Students Learn the Fundamental Exact Unification of Gravity, Relativity, Quanta and Elementary Charge

Hans-Otto Carmesin*

*[1]Gymnasium Athenaeum, Harsefelder Straße 40, 21680 Stade
[2]Studienseminar Stade, Bahnhofstr. 5, 21682 Stade
[3]Universität Bremen, Fachbereich 1, Postfach 330440, 28334 Bremen
hans-otto.carmesin@t-online.de

Abstract

Near Earth Orbit asteroids require a precise orbit determination. For it, in principle, essential theories are quantum physics, QP, and general relativity, GR. However, these are not fully compatible. How can this be overcome? For it, the volume dynamics, VD, of intergalactic space and of volume in nature in general are derived. The VD provide QP and GR as well as gravity. Moreover, the VD are tested at fundamental problems: The VD provide the dark energy. The VD solve the Hubble tension and predict H_0-values ranging from redshift $z = 1090$ to $z = 0$. The VD solve the cosmological constant problem. Thus, the VD bridge cosmology, QP and GR and provide a deep insight. In a didactic analysis, all results are derived according to the hypothetic deductive method, and all didactic steps are elaborated. So you can directly use the concept for your courses. The learning process has been tested in various learning groups, and experiences are reported.

1. Introduction

1.1. Comets: Beautiful and possibly dangerous

In July 2020, the comet Neowise had a distance to Earth of only $103 \cdot 10^6$ kilometers (Fig. 1). At April 13th in 2029, the asteroid Apophis will pass Earth at a distance of 35 500 km, see Bancelin et al. 2012. So, a defence of Earth is analysed. For it, a precise observation of the time and position of asteroids is essential: Atomic clocks and curvature of space and time are relevant. These are described by two essential physical theories: quantum physics, QP, and general relativity, GR. So, a fundamental theory providing QP and GR is essential. Such a theory is proposed. As a test, it solved the Hubble tension and the cosmological constant problem, Carmesin (2024g).

Fig. 1: Comet Neowise near Hamburg at August 2020.

Fig. 2: Wavelengths of light can be measured. This photo of a hydrogen lamp is taken by a smartphone camera with a cross-grating in front of the lens.

1.2. Experiments showing quanta and redshift

At a measurable wavelength λ of light, a smallest portion of energy can be measured with help of λ, the velocity of light c and the Planck constant h. as follows (Fig. 3): $E = h \cdot \frac{c}{\lambda}$ {1}

Fig. 3: A LED is illuminated and provides the voltage $U = 2.137$ V and electrons with the energy $E = 2.137$ eV.

The experiments in Figs. (2,3) represent an example of the quantization of energy. At a distance r from a mass M, light has a gravitational redshift: Light starts with a wavelength λ_∞ at a very large distance r_∞. At r_∞, the gravitational field is negligible. When the light arrives at a distance r from M, it has the wavelength $\lambda(r)$: $\lambda(r) = \frac{\lambda_\infty}{\sqrt{1-R_S/r}}$ with $R_S = \frac{2 \cdot G \cdot M}{c^2}$ {2}

G is Newton's gravitational constant, R_S is the Schwarzschild radius. So, the periodic time of such light is the following function: $T(r) = \frac{T_\infty}{\sqrt{1-R_S/r}}$ {3}

Such periodic time is used in optical atomic clocks (Gill et al. (2008) and Fig. 4). In traditional physics, light is used as a measure of space. Thus, the gravitational redshift is an example for the curvature of space, see Einstein (1915).

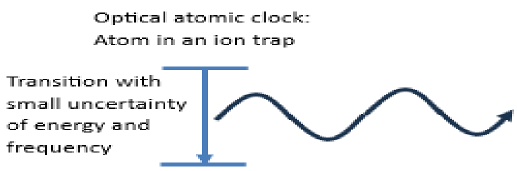

Fig. 4: In an optical atomic clock, an atom emits a photon, with T, with $\frac{\Delta T}{T} = 10^{-18}$, see Huntemann et al. (2016).

1.3. **Traditional concept of physics**

In present – day physics, the experiments in section (1.1) are interpreted as follows: Objects such as light or atoms move and interact in a metric space, described by general relativity, see e. g. Einstein (1915), Delva (2018). Thereby, many objects are quantized at the microscopic level., see e. g. Planck (1900), Einstein (1905), Heisenberg (1925), Schrödinger (1926), Ballentine (1998). However, QP and GR appear to be hardly compatible, see Einstein et al. (1935), Einstein (1948), Ballentine (1998), Nobbenius (2006). So, the foundations of QP and GR show problems: the cosmological constant problem (Nobbenius 2006), the Hubble tension (Riess 2022), nonlocality (Einstein et al. 1935). As we live in one world, a unification of GR, QP is important. For it, epistemology is essential:

1.4. **Epistemology**

Kircher, Girwidz und Häußler (2001, section 4.1.2) describe the hypothetic deductive method. In the epistemological literature, this method is also called hypothetico-deductive testing (Niiniluoto, Sintonen, Wolenski 2004, S. 214). The method consists of three steps: In the hypothetic step, a thesis or hypothesis is suggested for testing. In the deductive step, implications are derived. In the third step, the implications are compared with observation. Hereby, in principle, a falsification should be possible. This method is used here as well as in Carmesin (2024a-g, 2019, 2020, 2021 a-d, 2022 a-c, 2023 a-c).

1.5. **Volume in nature is a fundamental concept**

Volume in nature is fundamental, as objects exist, move and interact in the volume in nature. The hypothesis in section (1.4) consists of properties of the volume in nature. These properties are evident part of present-day knowledge. In the deductive step, the dynamics of volume in nature are derived, the volume dynamics, VD.

2. **Didactic and physical analysis**

In this section, the evident properties of volume in nature are presented (section 2.1) and essential implications are derived (section (2.2) in a self-contained manner. Moreover, didactic steps are developed. Furthermore essential achieved insights are reflected.

2.1. **Volume in nature: evident properties**

2.1.1. **Volume of intergalactic space**

2.1.1.1. **Physical analysis**

Volume in nature occurs in a relatively pure form in the intergalactic space. Volume $V_{intergalactic\ space}$ of intergalactic space, see Karttunen (2007):

$$V_{intergalactic\ space} \approx V \qquad \{4\}$$

2.1.1.2. **Didactic analysis**

In a first didactic step, we realize that traditional physics (section 1.2) uses different concepts of vacuum (Einstein 1917, Casimir 1948, Ballentine 1998, Hobson 2006, Nobbenius 2006).

In didactic step two, we realize that traditional physics applies the concept of a metric space. Volume-portions, propagating in different directions, are not considered (Einstein 1915, Hobson 2006).

In a 3^{rd} step, the meaning of volume in nature is clarified with help of the example of intergalactic space in a direct and general manner, that is not restricted to a particular theory. This example and the above two didactic steps have no essential learning barrier.

2.1.2. **Evident properties**

2.1.2.1. **Physical analysis**

In traditional physics and in metric space, the volumetric property of volume is characterized:

Increments of volume that are used in differential geometry and general relativity are analysed as follows, see Lee e. g. (1997), Hobson (2006) or Carmesin (2020): $dV_L = \prod_j^D \sqrt{|g_{jj}|} \cdot d\xi^j$ {5}
Hereby, D is the dimension of the analysed space or spacetime, the $d\xi^j$ are increments of locally orthogonal coordinates, and g_{jj} are the elements of the metric tensor. The subscript L specifies that the light – travel distance d_{LT} is used as a distance measure, see e. g. Einstein (1905), Hobson (2006), Condon (2018). Thereby, volume has basic properties that are provided by evident present-day knowledge about volume and electromagnetic waves: Firstly, volume and time are fundamental. Secondly, volume in nature has the volumetric property, see Eq. {5}. Thirdly, volume in nature has zero rest mass:

$$m_{vol,0} = 0 \qquad \{6\}$$

Fourthly, at a global level, volume is isotropic. This is at least a good approximation, see Carmesin (2023a). Fifthly, a basic property of electromagnetic waves is used: Electromagnetic waves exhibit the property of linear superposition. With it, special relativity, SR, has been derived, see e. g. Carmesin (2019, 2020, 2022a section 7.8).

2.1.2.2. **Didactic analysis**

In step one, the volumetric property is described by the metric theory via the metric tensor. This theory is based on differential geometry, see e. g. (Lee (1997), so it has a robust mathematical basis. And the metric theory of space is also used by traditional physics, so there is a maximum of continuity with the traditional physics (section 1.2). This traditional theory has no essential learning barrier. In step two, properties {1} to {5} are introduced. These are evident.

2.2. **Implications of the evident properties**

The evident properties of volume in nature (section 2.1) imply essential results. These are derived in the deductive step of the epistemological method:

2.2.1. Portions of volume

2.2.1.1. Physical analysis

Theorem: Law of the existence of volume-portions in nature. Firstly, volume in nature propagates at the velocity of light. Secondly, volume in nature consists of volume-portions, VPs. Proof: see Carmesin(2024g).

2.2.1.2. Didactic analysis

In a first didactic step, the following is shown: $m_{vol,0} = 0$ implies that volume-portions propagate at the velocity of light. This step has no mathematical learning barrier. However, this step provides a mental learning barrier: It is shown that volume-portions move at $v = c$, this differs from usual metric space. In a second step, it is shown that the average velocity of VPs is zero, as VPs are isotropic. This step has no special learning barrier. In a third step, it is shown that the average zero requires several VPs. This step has no special learning barrier.

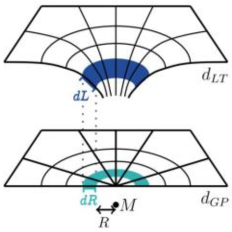

Fig. 5: In the vicinity of a mass M or effective mass M_{eff}, the radial increment dL of the light travel distance d_{LT} is increased with respect to the original increment dR that would occur in the limit M to zero. This increment dR is called gravitational parallax distance d_{GP}.

2.2.2. Measurable gravitational parallax distance

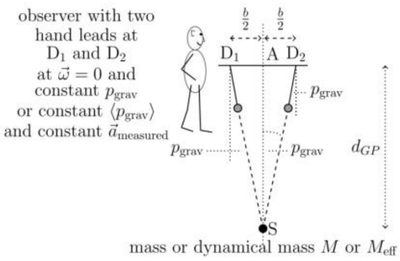

Fig. 6: Measurement of gravitational parallax distance d_{GP} via the angle of gravitational parallax p_{grav}: D_1, D_2 and S form an isosceles triangle D_1D_2S. The baseline D_1D_2 has the centre A. An effective mass M_{eff} is measured at constant measured acceleration $\vec{a}_{measured}$.

2.2.2.1. Physical analysis

It is fundamental that the distance d_{LT} in curved space and the corresponding distance d_{GP} in flat space can be measured simultaneously in the same curved space (Fig. 5). Firstly, it is well-known how the light travel distance can be measured, see e. g. Hobson (2006), Condon and Matthews (2018). Secondly, the gravitational parallax distance d_{GP} is founded by defining corresponding measurement procedures: We describe how an observer can apply two hand leads, in order to measure the distance to an object (Fig. 6). Definition 1: Gravitational parallax distance to a mass: The gravitational parallax distance, d_{GP}, between an observer and a (dynamical) mass is defined by the measurement procedure in Fig. (6): $d_{GP} = \frac{0.5b}{tan(p_{grav})}$ {7}

For details, see Carmesin (2023a,2024g)

2.2.2.2. Didactic analysis

In a first step, the measurement procedures are defined. Hereby, the measurement of a distance by triangulation is a well-known procedure. Similarly, hand leads are well-known devices. Additionally, it is clear that hand leads are influenced by an acceleration and by a rotation. Thus, these are excluded with respect to a reference mass M or M_{eff}. This step has a medium-sized learning barrier: The acceleration and angular velocity must be set to zero with help of a closed loop control. In a second didactic step, the following is reflected: In general, M or M_{eff} can be accelerated with respect to some other object. Accordingly, the gravitational parallax distance d_{GP} is measured relative to M or M_{eff}. This relation is not very complex, and so it can be understood clearly. Moreover, the fact that d_{GP} is measured relative to M or M_{eff} is clear from the very beginning. Thus, this step has only a medium-sized mental learning barrier. Altogether, only medium-sized learning barriers occur in the measurement procedures for d_{GP}. Moreover, the d_{GP} provides a useful tool: Curved space can be compared with flat space by observation. With it, the dynamics of distances and of volume can be measured and analysed.

2.2.3. Representation of space with help of VPs

2.2.3.1. Physical analysis

In general, space can exhibit curvature (Fig. 5). It can be represented with help of a metric tensor g_{ij}. However, as space consists of volume portions, it is valuable to transform the description with the metric tensor to a description using VPs. This transformation is worked out in this section:

A mass M causes additional volume δV. It can be described with help of the metric tensor, see e. g. Hobson (2006, section 2.10):

$\delta V = \prod_j^D \sqrt{|g_{jj}|} \cdot d\xi^j - \prod_j^D \sqrt{|g_{jj,flat}|} \cdot d\xi^j$ {8}

Hereby, $g_{jj,flat}$ is the metric tensor of flat space, whereas g_{jj} is the metric tensor of curved space.

At each point in D-dimensional space or spacetime, a local orthogonal coordinate system can be used, Lay (2016, section 6.4).

We name the corresponding incremental orthogonal coordinates $d\xi^j$ and the corresponding basis vectors $d\vec{\xi}^j$. Hereby, in traditional general relativity, GR, two types of components are used, contravariant components are marked by an upper index and covariant components are marked by a lower index, see e. g. Landau and Lifschitz (1971, § 6). The geometric properties of D-dimensional space or spacetime can be described by the metric tensor g_{ij},

see e. g. {Landau and Lifschitz (1971, § 84).Thereby, the metric tensor can be obtained by the scalar product of the basis vectors, see e. g. Hobson (2006, sections 4.1 – 4.4): $g_{ij} = d\vec{\xi}_i \cdot d\vec{\xi}_j$ {9}
It is useful to represent the light-travel distance based length of each basis vector $d\vec{\xi}_{L,j}$ by an increment $dx_{L,j}$ which subsumes the root of the element of the metric tensor, see e. g. Hobson (2006, section 2.10) and Fig. (7): $dx_{L,j} = \sqrt{g_{jj}} \cdot d\xi^j$ {10}
Consequently, the volume spanned by the D basis vectors is as follows, see e. g. Hobson (2006, section 2.10): $dV_L = \prod_j^D dx_L^j = \prod_j^D d\xi_L^j \cdot \sqrt{|g_{jj}|}$ {11}

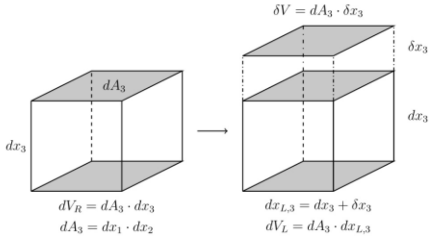

Fig. 7: VPs can be represented with cubes and cuboids.

Theorem: Law of representation by additional volume (proof: Carmesin 2024g): At each point, the metric tensor can be represented by differences and relative differences as follows: Firstly, even if the space is curved, at each point, the incremental orthogonal coordinates of flat space dx_j can be measured. These are related to the metric tensor of flat space $g_{ij,flat}$ as follows: $dx_j = \sqrt{g_{jj,flat}} \cdot d\xi^j$ {12}
Secondly, at each point in space, the length difference caused by curvature can be measured:
$\delta x_j = dx_{L,j} - dx_j$ {13}
Thirdly, at each point in space, the incremental volume in flat space can be measured:
$dV_R = \prod_j^D dx^j = \prod_j^D d\xi^j \cdot \sqrt{|g_{jj,flat}|}$ {14}
Fourthly, at each point in space, the additional volume can be measured: $\delta V = dV_L - dV_R$ {15}
Fifthly, at each point, the intensive quantity, normalized by the advantageous d_{LT} based volume dV_L, can be measured: $\varepsilon_L := \frac{\delta V}{dV_L}$ {16}
That ratio is named relative additional volume ε_L. Sixthly, at each point, the relative additional volume is the following function of dV_L and dV_R:
$\varepsilon_L = 1 - \frac{dV_R}{dV_L}$ {17}

2.2.3.2. **Didactic analysis**

In a first didactic step, the increments of a VP are described with help of the metric tensor, see Eqs. {9} and {10}. This notation is especially valuable for readers familiar with the metric tensor notation. This step has no special learning barrier for such learners. In a second didactic step, the root of the respective metric tensor element is subsumed multiplied by the increment of the coordinate of the orthogonal coordinate system, see Eq. {11} and Fig (7). The product is an increment $dx_{L,j}$. The resulting notation is especially valuable for readers familiar with Cartesian coordinate systems. This step has only a medium-sized mental learning barrier for readers that stick to one of the two notations. In didactic step three, additional volume is introduced: Based on the volumetric property of volume in nature and on the measurements of the light-travel distance and the gravitational parallax distance, the additional volume caused by a mass M or M_{eff} is derived in a direct manner. This step is algebraically straight forward. This step has only a medium-sized mental learning barrier for readers that stick to a particular personal view in which difference of volume-portions are avoided artificially. In a fourth didactic step, a very useful intensive quantity is derived. This step is algebraically straight forward. This step has a medium-sized mental learning barrier for readers that do not realize the immense advantage of intensive physical quantities with respect to the search of universal laws of physics. This step has another medium-sized mental learning barrier for readers that do not realize that the relative additional volume should have the light-travel distance based volume in the denominator in order to achieve universal local laws of physics. In a didactic fifth step, some algebraic transformations are used in order to prove the theorem. This step has no special learning barrier. Altogether, the five didactic steps have no special algebraic or formal or geometric learning barrier, and they have several mental learning barriers. Moreover, the relative additional volume has many valuable implications. This topic provides a high learning efficiency, deep insights and a useful tool.

2.2.4. **Differential equation of volume**

Portions of additional volume propagate with $v = c$ (section 2.2.1).

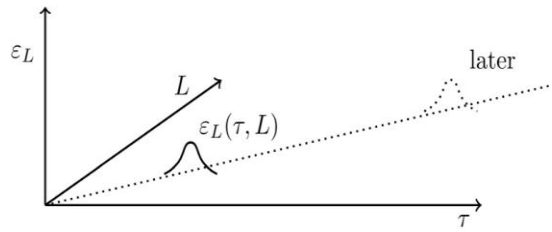

Fig. 8: A localizable portion of relative additional volume ε_L propagates in space. The relative additional volume is analysed as a function of τ and $\vec{L}$.

2.2.4.1. **Physical analysis**

The volume-portions δV in sections (2.2.1, 2.2.3) have a valuable completely new property: They can propagate in space: Theorem: Law of propagation of relative additional volume (proof: Carmesin 2024g): Localizable VPs propagate as follows: Firstly, during an increment of time $d\tau$, the local maximum of relative additional volume ε_L changes its position by a spatial increment $d\vec{L}$ as follows:
$d\vec{L} = \frac{\partial \vec{L}}{\partial \tau} d\tau$ with $\frac{\partial \vec{L}}{\partial \tau} = c\vec{e}_v$ {18}
Secondly, if the relative additional volume ε_L is analysed as a function of τ and $\vec{L}$ (Fig. 8), then it fulfils

the following differential equation, DEQ of VD:

$$\frac{\partial}{\partial\tau}\ \varepsilon_L = -v \cdot \vec{e}_v \cdot \frac{\partial}{\partial\vec{L}}\ \varepsilon_L \quad \text{with} \quad v = c \qquad \{19\}$$

Thirdly, in principle, there is no difference between a portion of additional volume δV and a localizable VP. Consequently, Eqs. {18} and {19} hold for each localizable VP. Fourthly, each localizable volume-portion propagates according to the following Lorentz invariant DEQ: $\dot{\varepsilon}_L^2 - c^2 \cdot \left(\frac{\partial}{\partial\vec{L}}\ \varepsilon_L\right)^2 = 0$ {20}

2.2.4.2. **Didactic analysis**

In a first didactic step, the motion of the maximum is analysed. As a localizable portion of relative additional volume propagates at $v = c$, the position $\vec{L}$ of its maximum value moves according to Eqs. {18} and {20} as a function of time, $\vec{L} = \vec{L}(\tau)$. This motion is similar to the motion of a point-like mass. Thus, this step provides no essential learning barrier for learners familiar with analysis. In a second didactic step, the fact is realized that the portion is not point-like. Accordingly, its form is described as a function of τ and $\vec{L}$ (Fig. 8). This step and this function $\varepsilon_L(\tau, \vec{L})$ provide no essential learning barrier for learners that are familiar with distribution functions or with density functions or with atomic orbitals. In didactic step three, both functions $\vec{L}(\tau)$ and $\varepsilon_L(\tau, \vec{L})$ are combined. For it, the fact is used that the maximum has zero derivative. With it, the DEQs {19} and {20} are derived. This derivation uses well-known methods of analysis, so it provides no essential learning barrier. Hence, this step has no essential learning barrier.

2.2.5. **Schrödinger equation derived from VD**

2.2.5.1. **Physical analysis**

Theorem: Law of the derived GSEQ: Eq. {19} implies the generalized SEQ, GSEQ (proof: Carmesin 2024g): $i\hbar \frac{\partial}{\partial\tau}\ \Psi = c \cdot \hat{\vec{p}}\Psi = \hat{H}\Psi$ {22}

Hereby, the wave function is $\Psi = t_n \cdot \dot{\varepsilon}_L$. And the normalization factor is t_n.

Theorem: Law of the derived SEQ: Eq. {22} implies the SEQ (proof: Carmesin 2024g): In the limit of slow objects, the SEQ proposed or postulated by Schrödinger is derived:

$$i\hbar \frac{\partial}{\partial\tau}\Psi \doteq \frac{\hat{p}^2}{2m_0}\Psi + E_{pot}\ \Psi = \hat{H}\Psi \qquad \{23\}$$

2.2.5.2**. Didactic analysis**

In a first didactic step, the GSEQ is derived from the DEQ {19} of volume. For it, a time derivative is applied and $i\hbar t_n$ is multiplied. This step has no essential analytic, algebraic or geometric learning barrier. The identification of DEQ {22} with a GSEQ represents a mental learning barrier for all readers that expect the form of the SEQ proposed by Schrödinger (1926). That learning barrier is reduced with help of a second didactic step: The traditional wave function is identified, see DEQ {23}. This step has only a mental learning barrier for those readers that have the opinion it would be impossible to identify the physical meaning of the wave function. In fact, this opinion is quite common, see e. g. Kumar (2018, p. 14). However, that opinion has never been proven in general. Fortunately, some authors regard the interpretation of the wave function as an open question, see e. g. Scheck (2013, p. vii, sections 1.3-5.1). That learning barrier is reduced further by using the traditional operators of momentum and energy in a third didactic step, see DEQs {22,23}. This step has only a mental learning barrier: The Hamiltonian of the GSEQ describes relativistic objects, including the option of a nonrelativistic limit. In contrast, the SEQ proposed by Schrödinger describes non-relativistic or slow objects only. Thus, the traditional SEQ is a special case of the GSEQ. That learning barrier is reduced further by deriving the non-relativistic limit. The result is the traditional form of the SEQ. This step has no essential learning barrier. Altogether, the derivation provides great insights: It bridges QP and cosmology. It clarifies the meaning of the wave function. It provides a generalization.

2.2.6. **Generalized potential**

2.2.6.1. **Physical analysis**

The law of propagation of localizable relative additional volume can be applied to the vicinity of a mass M or effective mass: Theorem: Law of the derived generalized gravitational interaction. In the vicinity of a mass M or effective mass M_{eff}, the relative additional volume ε_L exhibits the following properties (proof: Carmesin 2024g, we use spherical polar coordinates with M at the origin, $d_{GP} = dR$ and $d_{LT} = dL$.): (1) The relative additional volume ε_L propagates according to Eq. {19}. That Eq. is multiplied by c: $c \frac{\partial}{\partial\tau}\ \varepsilon_L = \vec{e}_v \cdot \frac{\partial}{\partial\vec{L}} \cdot (-c^2 \cdot \varepsilon_L)$ {24}

The bracket in the above DEQ has the form of a generalized potential Φ_{gen}: $\Phi_{gen} := -c^2 \cdot \varepsilon_L$ {25}

(Hereby, the potential is generalized as it describes volume, whereby volume can generate matter in a phase transition, see Higgs 1964). The negative gradient of that generalized potential is the generalized field $\vec{G}_{gen}$:

$$\vec{G}_{gen} := -\frac{\partial}{\partial\vec{L}}\ (-c^2 \cdot \varepsilon_L) = -\frac{\partial}{\partial\vec{L}}\ \Phi_{gen} \qquad \{26\}$$

The DEQ {22} takes the form of the following rate gravity relation:

$$c \frac{\partial}{\partial\tau}\ \varepsilon_L = \vec{e}_v \cdot \frac{\partial}{\partial\vec{L}} \cdot \Phi_{gen} = -\vec{e}_v \cdot \vec{G}_{gen} \qquad \{27\}$$

(2) That rate gravity relation can be expressed with help of the following rate gravity scalar RGS_{gen}:

$$RGS_{gen} := \left(c \frac{\partial}{\partial\tau}\ \varepsilon_L\right)^2 - \vec{G}_{gen}^2, \quad \text{thus} \qquad \{28\}$$

$$RGS_{gen} = \left(c \frac{\partial}{\partial\tau}\ \varepsilon_L\right)^2 - \sum_j^D G_{gen,j}^2 \quad \text{and} \qquad \{29\}$$

$$RGS_{gen} = \left(c \frac{\partial}{\partial\tau}\ \varepsilon_L\right)^2 - \left(c \frac{\partial}{\partial\vec{L}}\ \Phi_{gen}\right)^2 \quad \text{and} \qquad \{30\}$$

$$RGS_{gen} = 0 \qquad \{31\}$$

(3) The generalized field is proportional to $\frac{1}{R^{D-1}}$:

$$|\vec{G}_{gen}| = \frac{1}{R^{D-1}} \qquad \{32\}$$

2.2.6.2. **Didactic analysis**

In a first didactic step, the form of DEQ {19} of VD provides a generalized potential $\Phi_{gen}(R)$ and field $\vec{G}_{gen}(R)$ of the volume in an exact manner. This step has no analytic or algebraic learning barrier for all learners familiar with fields and potentials. Even though the derived results are exact, there remains an open question about the interpretation – it will be answered in the next section. In didactic step two, a coordinate invariant scalar is derived. This step has no learning barrier for learners familiar with four-vectors. In a third didactic step, it is shown that the generalized field is proportional to $\frac{1}{R^{D-1}}$, whereby R is the radial coordinate or the gravitational parallax distance to the field generating mass M. This step has no special learning barrier. The derived potential, field and the proportionality to $\frac{1}{R^{D-1}}$ are very valuable, as they are exact. In contrast, the potential in Newton's theory of gravitation is not exact, see e. g. Hobson (2006).

2.2.7. **General relativity and gravity**

2.2.7.1. **Physical analysis**

We use spherical polar coordinates with radial coordinate R. The inverse root of g_{RR} of the metric tensor is named position factor ε_E: $\frac{1}{\sqrt{g_{RR}}} =: \varepsilon_E$ {33}

In general, ε_E is a function of R.

Theorem: Law of the derived curvature and interaction (proof: Carmesin 2024g): ε_E has the following properties: $\varepsilon_E = 1 - \varepsilon_L$ {34}

The generalized field is proportional to the mass M and to $\frac{1}{R^{D-1}}$: $\left|\vec{G}_{gen}(R)\right| \propto \frac{M}{R^{D-1}}$ {35}

The proportionality factor is interpreted as a universal constant of nature G_{gen}. It must be obtained from observation: $\left|\vec{G}_{gen}(R)\right| = G_{gen} \cdot \frac{M}{R^{D-1}}$ {36}

$\varepsilon_E(R)$ fulfils: $\frac{G_{gen}M}{c^2 R^{D-1}} = \varepsilon_E \frac{\partial \varepsilon_E}{\partial R}$ {37}

That DEQ and the relation $lim_{R\to\infty}\varepsilon_E = 1$ imply:

$$\varepsilon_E(R) = \sqrt{1 - \frac{2G_{gen}M}{c^2} \cdot \frac{1}{R^{D-2}(D-2)}} \quad \{38\}$$

Observation shows: $G_{gen}(D = 3)$ is Newton's constant of gravitation: $G_{gen}(D = 3) = G$ {39}

and $G_{gen} = G \cdot L_P^{D-3} \cdot (D-2) = G_D$ {40}

Hereby, L_P is the Planck length:

$$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \cdot 10^{-35}\text{ m} \quad \{41\}$$

At $D = 3$, the Schwarzschild radius $R_S = \frac{2GM}{c^2}$ implies: $\varepsilon_E(R) = \sqrt{1 - \frac{R_S}{R}}\quad at \quad D = 3$ {42}

At a dimension $D \geq 3$, the Schwarzschild radius is as follows, see e. g. Carmesin (2019, section 2.6):

$$R_{SD} = (R_S \cdot L_P^{D-3})^{\frac{1}{D-2}} \quad \{43\}$$

so, $\varepsilon_E(R) = \sqrt{1 - \left(\frac{R_{SD}}{R}\right)^{D-2}}$ at $D \geq 3$ {44}

As a consequence, in three-dimensional space, the generalized field is equal to the gravitational field:

$$\vec{G}_{gen}(R) = -\frac{GM}{R^2} \cdot \vec{e}_L = \vec{G}^*(R) \quad \text{at} \quad D = 3 \quad \{45\}$$

Hereby, $\vec{e}_L = \vec{e}_v$. Consequently, at $D \geq 3$:

$\vec{G}_{gen}(R) = -\frac{G_D M}{R^{D-1}} \cdot \vec{e}_v$, thus

$$\vec{G}_{gen}(R) = -\frac{G \cdot M}{R^2} \cdot \vec{e}_v \cdot (D-2) \cdot \left(\frac{L_P}{R}\right)^{D-3} \quad \{46\}$$

2.2.7.2. **Didactic analysis**

In a first step, the position factor is used as an abbreviation. Moreover, the relative additional volume is related to the position factor. This step has no special learning barrier. That step provides the valuable insight that the relative additional volume explains the position factor and the metric tensor element g_{RR}. In a second didactic step, a term for the generalized field is derived. This step has no special learning barrier. In step three, the DEQ for the position factor is derived. Moreover, the solution of that DEQ is derived. This step has no special learning barrier. In a fourth didactic step, it is shown that Newton's constant of gravitation applies to the generalized potential and field. Thereby, it is shown that the derived position factor explains the element g_{RR} of the metric tensor in general relativity. With it, the other elements of the metric tensor of the Schwarzschild solution can be derived, if desired, see e. g. Carmesin (2023a). This step has no special learning barrier. In a fifth step, the exact gravitational field is derived. This step has no special mathematical learning barrier. The results of the above didactic steps are exact, insightful, useful and general, as they show that the same volume dynamics in the DEQ {19} provide the SEQ as well as the Schwarzschild metric of GR as well as the exact gravitational field. Moreover, the results are general, as they can be applied to the vicinity of each effective mass. More generally, the relation to GR is provided in Carmesin (2024g).

2.2.8. **Local formation of volume, LFV**

2.2.8.1. **Physical analysis**

As a byproduct of the propagation of relative additional volume, there can occur local formation of volume, LFV: Definition: LFV: If additional volume $\underline{\delta} V_{jj}$ forms in a volume dV_L and in a direction j during a time $\underline{\delta}\tau$, then this process can be described by the following normalized rate of unidirectional LFV, see Fig. (7): $\dot{\underline{\varepsilon}}_{L,jj} := \frac{\underline{\delta} V_{jj}}{\underline{\delta}\tau \cdot dV_L}$ {47}

In the vicinity of a mass M or an effective mass M_{eff}, and at a d_{GP} based distance R from M or M_{eff}, the following holds for the normalized rate:

Theorem: Law of locally formed volume, LFV:

(1) In the far distance approximation, FDA, the ratio $\frac{R_S}{R}$ is relatively small. At first order in that ratio $\frac{R_S}{R}$, the normalized rate is: $\dot{\underline{\varepsilon}}_{L,jj}^2 c^2 = G_{gen,j}^2$ {48}

$G_{gen,j}$ is the component j of the generalized field, see section (2.2.6). The full additional volume is obtained by the sum with respect to the components j,

as nondiagonal components $\underline{\dot{\varepsilon}}_{L,ij}$ do not provide additional volume: $\underline{\dot{\varepsilon}}_L^2 c^2 := \sum_j \underline{\dot{\varepsilon}}_{L,jj}^2 c^2 = \sum_j G_{gen,j}^2$ {49}
This relation can be expressed in a Lorentz invariant form: $\underline{\dot{\varepsilon}}_L^2 c^2 - \sum_j G_{gen,j}^2 = 0$ {50}
Hereby, the generalized field is equal to the exact expression of the gravitational field, whereas Newton's gravitational field is an approximation, see sections (2.2.6) and (2.2.7).

2.2.8.2. **Didactic analysis**

In a first step, the fact of formation of additional volume is realized. This step has no special learning barrier, as the dependence of R is obvious. In step two, the rate of LFV is derived. As the derivation is quite simple, there is no special learning barrier.

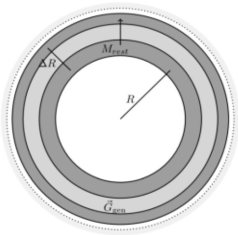

Fig. 9: A mass M (dark grey) in a shell at a radius R is lifted to a radius $R + \Delta R$: Differential parts dM are lifted, while the rest M_{rest} is still at R. Thereby the field $\vec{G}_{gen}$ (medium grey) in the shell with radius R and thickness ΔR becomes zero, when the whole mass is at $R + \Delta R$.

2.2.9. **Energy density of the field**

2.2.10.1. **Physical analysis**

Theorem: Law of the energy density of a gravitational field (Proof: Carmesin 2024g or 2023a):
(1) A gravitational field $\vec{G}_{gen}$ has the following energy density: $u_{gr.f.} = -\frac{|\vec{G}_{gen}^2|}{8\pi G}$ {51}
(2) A gravitational field $\vec{G}_{gen}$ causes LFV with a rate: $\underline{\dot{\varepsilon}}_L^2\, c^2 \doteq \sum_j G_{gen,j}^2 = \vec{G}_{gen}^2$, {52}
at first order in the FDA. At that locally formed volume, there occurs the following density of a generalized kinetic energy: $u_{gen,kin} = \frac{\underline{\dot{\varepsilon}}_L^2 \cdot c^2}{8\pi G}$ {53}
Similarly, the analogous result can be derived for the relative additional volume: $u_{gen,kin} = \frac{\dot{\varepsilon}_L^2 \cdot c^2}{8\pi G}$ {54}

2.2.10.2. **Didactic analysis**

In a first didactic step, the process in Fig. (9) is introduced. As this is very intuitive, there is no special learning barrier. In step two, the change of generalized potential energy in the process in Fig. (9) is derived. As the generalized potential is exact, that derivation is exact. This increases the confidence. As the mathematical steps involve at most a simple integration, this step has no special learning barrier. In step three, the sign of the potential energy is derived. As the transfer of energy in the process in Fig. (9) is transparent, there is no special learning barrier. This is insightful, as it shows that the sum of the energy densities is zero. In step four, the LFV is applied. With it, the energy density of the gravitational field is identified directly. On that basis, the positive term in Eq. {53} is directly identified with another energy density. According to its sign and its form, it is interpreted as a generalized kinetic energy density. As the interpretation does not require a proof, this step has no special learning barrier.

3. Experience: learning process and learners

The experiences with learning groups have been documented in terms of photographs of the blackboard and with help of additional reports. These are summarized as follows. In a general studies course at the university, the learning process was enriched by a permanent discussion of the achieved results and by exercises about the derived relations. In particular, the learning process took place in eight unites, each lasting 90 minutes: (1) An introduction, the epistemological method, the volume in nature, the evident properties and a summary of special relativity have been treated. (2) The existence of several volume-portions has been derived. The measurement methods for d_{LT} and d_{GP} have been introduced (Fig. 6). The resulting maps (Fig. 5) have been treated. The additional volume (Fig. 7) and the law of representation by additional volume have been derived. (3) The propagation of VPs has been derived (DEQ {19} and Fig. 8). The GSEQ and SEQ have been derived. In exercises, several solutions have been developed. (4) The stationary SEQ has been derived. In exercises, several solutions have been developed. And the semiclassical limit has been introduced. Hereby, the principle of least action has been developed. It has been shown, how the Einstein field equation has been derived with help of that principle. (5) Newtonian gravity was summarized, in order to prepare the introduction of the generalized field $\vec{\mathrm{G}}_{gen}$ and potential Φ_{gen}. Then, these quantities $\vec{\mathrm{G}}_{gen}$ and Φ_{gen} have been derived. (6) The position factor including the curvature of space have been derived. In exercises, examples have been analysed and the Heisenberg uncertainty principle has been investigated. (7) LFV has been derived. (8) The energy densities of the gravitational field and of the generalized kinetic energy have been derived.

A quantum gravity group of a research club meets 90 minutes each week: Topics such as quantum computers, cosmology, astrophysics or quantum gravity are treated. In that group, essentially the same learning process has been treated in an extra meeting for one and a half days at a weekend. Hereby, questions and exercises, including adequate derivations, have been treated. So, training, metacognitive activity and experience of self-efficacy and competence are provided.

4. Discussion

Comets and asteroids could collide with Earth. So, planetary defence is organized, see Michel (2016). Hereby, precise observations, QP and GR are important. But QP and GR are incompatible, see Einstein et al. (1935). Here, the VD is derived from evident properties. The VD implies and generalizes QP and GR. And the VD solves fundamental problems of QP and GR. And the VD have been tested: (1) The density of volume has been derived from the VD, see Carmesin (2023a, 2024c). (2) The VD solves the Hubble tension and predicts future measurements, see Carmesin (2023a, e, 2024d). (3) The VD solves the cosmological constant problem, see Carmesin (2023, a, 2024f,g). The results are in precise accordance with observation. Thereby, no fit has been executed and no postulate has been proposed. Such a testing of a (evident) hypothesis (section 1.4) and such a deduction from prior knowledge have a high learning efficiency, see Hattie (2006). The learning process uses everyday life contexts, so it is meaningful, see Muckenfuß (1995), and it achieves an additional high learning efficiency, see Hattie (2006).

The learning process of VD has been tested in two learning groups. That process takes 720 minutes at a university course as well as in a research club. In both learning groups, the students were able to perform exercises and to use instructions in order to derive parts of the theory. Moreover, the VD has been used in order to derive the elementary charge, see Carmesin (2021a, 2024g). This result is beyond GR, QP and the standard model of elementary particles.

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