Students discover the Schwarzschild metric at a free fall tower

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Abstract

In everyday life, time and space are essential. Moreover, space and time are fundamental concepts of physics. For it, Newton proposed flat space and time evolving at a constant rate as a basis. Accordingly to its relevance in everyday life and in physics, students are interested in the topic. Here, I present a learning process, by which learners can achieve the essential insights of special relativity and general relativity in an exact manner. Thereby, students experience basic principles directly at a free fall tower and by taking photographs in a school observatory. Using these basic principles, they achieve inspiring and exact results on their own, after an appropriate instruction. I present the learning process and a didactic analysis, so that you can directly use the concept for teaching. I tested the learning process in various learning groups, and I report about experiences.

1. Introduction

Newton (1687) postulated that space is flat and static and that time evolves at a constant rate. However, in their school observatory, students take photos of a gravitational lens, see Fig. 1 and Carmesin (2018a,b). With it they confirm the concept of curved spacetime proposed by Einstein (1915). But how can they understand or experience it?

1.1. Organization of the paper

We propose our learning process in section 2. In part 3, we provide a didactic analysis. Experiences with teaching are presented in part 4. We discuss our findings in section 5.

2. Learning process

The students are members of a research club. They are in classes 5 to 13. Accordingly, the younger pupils describe findings and graphic representations, intermediate students evaluate measured quantities, while advanced students provide derivations.

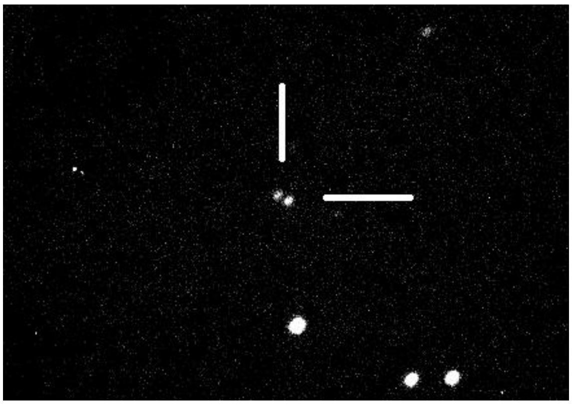

Fig.1: Students observe the twin quasar, a gravitational lens: It causes the two marked pictures of the same galaxy.

2.1. Special relativity

As a precondition of teaching, the students treated already basic facts in special relativity. The students use the light curve of a binary star in order to realize that the velocity of light in vacuum does not depend on the velocity of the light source, see Fig. 2. In particular, the light curve is regular, see Fig. 3. If the light emitted by the two stars would have different velocities as a consequence of the different velocities of the two stars, then the light curve would be very different from the observed light curve. In this manner, the students confirm on their own that the velocity of light in vacuum does not depend on the velocity of the source of the light. Accordingly, light is used as a measure for space and time. This is the main principle of special relativity.

Based on that principle, the students analyse the concept of the light clock in Fig. 4. The black box emits a laser beam. The beam propagates 0.6 m to the bottom. Then it is reflected and propagates back the box. After that process, the box indicates that the time

Fig.2: Students observed the star W Ursae Mayoris. It is a photometric binary. The students measured the light curve.

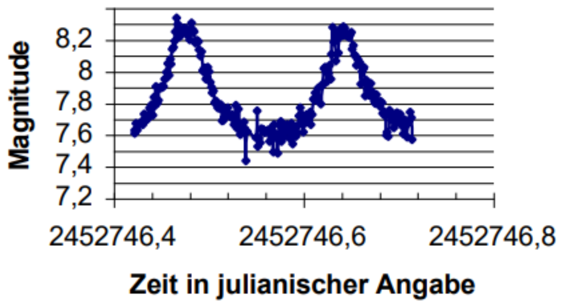

Fig.3: Students observed the star W Ursae Mayoris. It is a photometric binary. The students measured the light curve.

interval t = 4 ns has elapsed, as the light propagated the light-travel distance 1.2 m. However, if the clock moves at the velocity v = 0.18 m/ns, then an observer at rest observes that the light propagated the distance 1.5 m. Thus the observer at rest measures that the process lasted 5 ns, as the velocity of light has the same value in both systems or frames. Accordingly, between the same two events of the emission and arrival of the light at the box, there elapse two different amounts of time in the two different systems: In the own system of the clock, there elapses the shortest time $t_{own} = 4ns$. In the rest system or external system, there elapses the longer time $t_{ext} = 5ns$. Using the theorem of Pythagoras, the students derive the equation of the time dilation:

$$t_{ext} = t_{own} \cdot \gamma(v) \qquad \{1\}$$

Thereby, the Lorentz factor $\gamma(v)$ is defined as follows:

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \{2\}$$

Based on the Lorentz factor, the students analyse the energy $E(v)$ that a mass m_{own} or m_0 has at a velocity v:

$$E(v) = m_{own} \cdot \gamma(v) \cdot c^2 \qquad \{3\}$$

In the light clock, the velocity of the clock and the propagation of light are orthogonal to each other. Next, the students analyse the case of parallel velocity and propagation in Fig. 5. Thereby, they discover the length contraction.

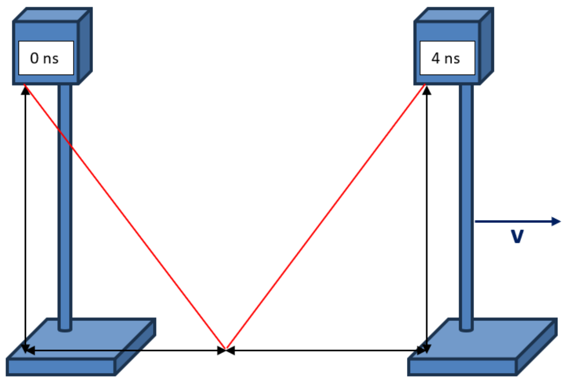

Fig.4: Light clock, drawn similar to Burisch et al. (2022).

Fig.5: An observer moving at the box with two mirrors measures the light-travel distance dr_{own} between the mirrors. An external observer at rest in his system r_{ext} measures the light-travel distance dr_{ext} between the same mirrors.

The observer at the moving box in Fig. 5 measures the following light-travel distance dr_{own}:

$$\frac{2}{c} \cdot dr_{own} = dt_{own} \qquad \{4\}$$

In his system r_{ext}, the observer at rest in Fig. 5 measures the following light-travel time: The propagation from the back mirror towards the front mirror requires the light-travel time $\frac{dr_{ext}}{c-v}$. The propagation from the front mirror towards the back mirror requires the light-travel time $\frac{dr_{ext}}{c+v}$. The observed light-travel time is the sum:

$$dt_{ext} = \frac{dr_{ext}}{c-v} + \frac{dr_{ext}}{c+v} = \frac{2}{c} \cdot dr_{ext} \cdot \gamma^2(v) \qquad \{5\}$$

The students use Eqs. {1}, {4} and {5}, in order to derive the relation:

$$dt_{ext} = dt_{own}\gamma = \frac{2}{c} dr_{own}\gamma = \frac{2}{c} dr_{ext}\gamma^2 \qquad \{6\}$$

This equation is solved for dr_{ext}. Thus, the students derive the following length contraction:

$$dr_{ext} = \frac{dr_{own}}{\gamma} \qquad \{7\}$$

The students realize that they derived the length contraction from the time dilation. Thus, these transformations are not independent from each other. Therefrom, they realize that space and time form a system. It is called spacetime.

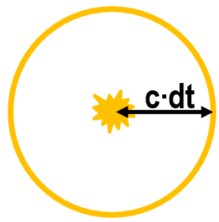

Fig.6: Light flash

Einstein (1905) described that system of spacetime with help of a light flash, see Fig. 6. Thereby, the light propagates the distance $dx^2 + dy^2 + dz^2$ during the scaled time $c^2 \cdot dt^2$. The difference is zero and denoted by the line element ds^2:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = 0$$

If that increment describes an own system moving at a velocity v in direction x relative to an external system, then the line element in the external system is as follows:

$$ds^2 = -\frac{c^2 dt^2}{\gamma^2} + dx^2 \cdot \gamma^2 + dy^2 + dz^2 = 0 \quad \{8\}$$

The line element describes a scalar product in four-dimensional spacetime.

The time dilation and the Lorentz factor are used in order to derive the relativistic energy of a mass m_0, for details of the derivation see Burish et al. (2022, pp 480-483):

$$E(v) = m_0 c^2 \cdot \gamma(v) \quad \{9\}$$

Alternatively, the relativistic energy can be obtained from observation, see Carmesin et al. (2020 p. 49).

2.2. **General relativity**

The students summarize: Special relativity is based on the invariance of the velocity of light. If an own system moves at constant velocity relative to an external system or rest system, then the values of physical quantities are transformed as a consequence of the invariance of the velocity of light. Such transformations are described by special relativity.

Students realize that we did not yet analyse consequences in accelerated systems. This analysis is the goal of general relativity. In order to investigate an accelerated system, the students make an excursion to a free fall tower. At that tower, the students can experience, measure and analyse consequences of a precisely known acceleration.

2.3. **Investigations at a free fall tower**

The students make an excursion to a free fall tower, see Fig. 7. At the tower, the students measure gravity and acceleration with their smartphones. For it, they can use the app phyphox or physics toolbox, for instance.

Fig.7: Free fall tower Scream at the Heidepark at Soltau

2.3.1. **Model experiment**

In order to understand the acceleration sensor, the students perform the model experiment in Fig. 8. A block represents the smartphone. A spring steel wire is attached to the block, and a mass is attached to the spring steel wire. This represents the acceleration sensor. At rest, the mass bends the spring steel wire downwards, and so it indicates the gravitational field $\vec{G}^*$ directed in the SAME direction (downwards), see Fig. 8.

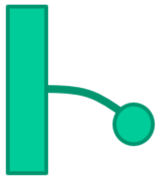

Fig.8: In this model experiment of an acceleration sensor, the sensor is at rest or accelerated upwards.

If the block experiences an acceleration $\vec{a}$ to the right, then the mass bends the spring steel wire to the OPPOSITE direction (to the left), see Fig. 9. Thus, at the display, the acceleration sensor shows the difference of acceleration observed by an external observer $\vec{a}_{ext}$ and gravitational field:

$$\vec{a}_{display} = \vec{a}_{ext} - \vec{G}^* \quad \{10\}$$

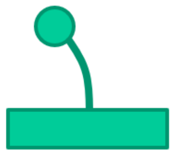

Fig.9: In this model experiment of an acceleration sensor, the sensor is accelerated to the right.

2.3.2. **Observations at the free fall tower**

At the free fall tower, the students put their smartphones into a shirt pocket, with the y-axis showing upwards. At the top of the tower, they start the recording of the y-component of the acceleration as a function of time.

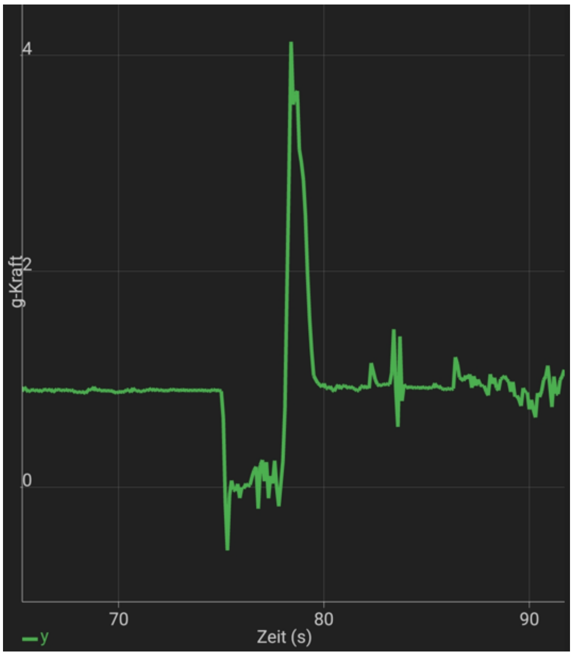

Fig.10: Measurement at the free fall tower with an acceleration sensor

A typical result is shown in Fig. 10. The students describe and explain the result as follows: At rest at the top of the tower, the acceleration is zero, so the sensor shows the absolute value of the gravitational field, see Fig. 8 or Eq. {10}:

$$\vec{a}_{display} = \vec{0}_{ext} - \vec{G}^* = |\vec{G}^*| = 9.81\,\frac{m}{s^2} \quad \{11\}$$

Most students prefer the explanation with Fig. 8.

At free fall, the displayed acceleration is zero, see Fig. 10. For it, most students prefer the explanation that the smartphone and the mass of the sensor fall in the same manner. As a consequence, the mass of the sensor does NOT bend the spring steel wire, see Fig. 11. Consequently, the displayed acceleration is zero. Some students explain the observation with Eq. {10}:

$$\vec{a}_{display} = \vec{a}_{ext} - \vec{G}^* = -9.81\,\frac{m}{s^2} + 9.81\,\frac{m}{s^2} = 0\,\frac{m}{s^2} \quad \{12\}$$

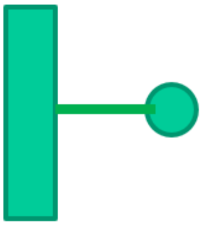

Fig.11: At free fall, the mass in the model experiment of an acceleration sensor does not bend the spring steel wire.

After free fall, the displayed acceleration is increases up to $40\,\frac{m}{s^2}$, see Fig. 10. Spontaneously, many students explain it with the slowdown of the gondola including the seats and passengers. After a short discussion, all agree that the display shows the sum of the acceleration of the slowdown and the gravitational field, according to Eq. {10} or Figs. 8 and 9.

2.4. **Equivalence principle**

When the students are reminded that we want to investigate the physics of the accelerated observer, they realize that the display shows the acceleration $\vec{a}_{own}$ of the accelerated observer in his own system. Accordingly, Eq. {10} is as follows:

$$\vec{a}_{down} = \vec{a}_{ext} - \vec{G}^* \quad \{13\}$$

In particular, if an observer is at free fall, then the acceleration in the own system is zero. Thus, the acceleration $\vec{a}_{ext}$ observed in the system of the Earth (or of the field generating mass in general) is equal to the gravitational field:

$$\vec{a}_{ext} = \vec{G}^* \quad \{14\}$$

In a discussion, the students realize that this is a basic physical principle, as it exactly describes the motion caused by a gravitational field. That principle is called equivalence principle.

2.5. **Principle of energy conservation**

In addition to the equivalence principle, we use the principle of energy conservation. Usually, no student asks whether that principle is applicable here. In order to provide a better overview, I proposed an analysis of the applicability of the principle of energy conservation.

Firstly, we realized that different energies are observed in different systems: For instance, if you ride a bicycle, then the kinetic energy is zero in your own system or frame. However, the kinetic energy is positive in the frame of an observer sitting at a bench nearby.

Secondly, we realized that the principle of energy conservation holds for a process taking place in a constant gravitational field: For instance, if you fall at your seat in the gondola of a free fall tower, then the height and the kinetic energy are both zero in your own frame. In contrast, the kinetic energy E_{kin} increases and the height as well as the potential energy E_{pot} decrease as a function of time in the frame of an observer sitting at a bench near the tower. Thereby, the absolute values of the changes ΔE_{kin} and ΔE_{pot} are equal:

$$\Delta E_{kin} = |\Delta E_{pot}| \quad \{15\}$$

Accordingly, the principle of energy conservation holds in both systems.

However, if the gravitational field would be switched off at the end of the process of acceleration, then the kinetic energy would still be positive, whereas the potential energy would be zero. Thus, the principle of energy conservation would not hold in the system at the bench. Of course, the gravitational field is constant, and as a consequence, the principle of energy conservation holds.

The students are informed: In each system that is invariant as a function of time, the principle of energy conservation holds. Emmy Noether (1918) derived this result in general.

2.6. **Derivation of the exact energy function**

In an interactive process, we analysed the energy of a mass m_0 that is at free fall in the gravitational field of a mass M and that has the velocity v = 0 in the radius r_∞ describing the limit $r \to \infty$, see Eq. {9}:

$$E(r_\infty, v) = m_0 \cdot c^2 \quad \{16\}$$

During free fall, v increases, so the energy is multiplied by Lorentz factor in {2}. According to Energy conservation, the Lorentz factor is compensated by a position factor $\varepsilon(r)$, so that the product of both factors is one:

$$1 = \gamma(v) \cdot \varepsilon(r) \quad \{17\}$$

We apply the derivative and the chain rule:

$$0 = \frac{d}{dr}\Big(\gamma\big(v(r)\big) \cdot \varepsilon(r)\Big) = \frac{d\gamma}{dv} \cdot \frac{dv}{dr} \cdot \varepsilon + \gamma \cdot \frac{d\varepsilon}{dr} \quad \{18\}$$

The two above factors of the position factor are evaluated with help of the chain rule:

$$\frac{dv}{dr} = \frac{dv}{dt} \cdot \frac{dt}{dr} = \frac{a}{v} \quad \& \quad \frac{d\gamma}{dv} = \gamma^3\,\frac{v}{c^2} \quad \{19\}$$

The derivatives in Eq. {19} are inserted in Eq. {18} (representing energy conservation):

$$\gamma^3 \frac{v}{c^2} \cdot \frac{a}{v} \cdot \varepsilon + \gamma \cdot \frac{d\varepsilon}{dr} = 0 \qquad \{20\}$$

In order to derive a differential equation, we solve for the derivative of the position factor. Hereby, we apply energy conservation in Eq. {17}:

$$\frac{d\varepsilon}{dr} = \varepsilon'(r) = -\frac{a}{c^2} \cdot \frac{1}{\varepsilon(r)} \qquad \{21\}$$

In order to relate the position factor to the gravitational field, we apply the equivalence principle. Hereby we remind that we did not explicate the sign of the acceleration:

$$\varepsilon'(r) = -\frac{|G^*|}{c^2} \cdot \frac{1}{\varepsilon(r)} = -\frac{GM}{c^2 \cdot r^2} \cdot \frac{1}{\varepsilon(r)} \qquad \{22\}$$

We abbreviate the mass M with help of the Schwarzschild radius:

$$R_S = \frac{2GM}{c^2} \qquad \{23\}$$

With it, the differential equation is as follows:

$$\varepsilon'(r) = -\frac{R_S}{2 \cdot r^2} \cdot \frac{1}{\varepsilon(r)} \qquad \{24\}$$

In order to obtain a solution, we make an Ansatz:

$$\varepsilon(r) = \sqrt{1 - \frac{R_S}{r}} \qquad \{25\}$$

We confirm the Ansatz by inserting it into the differential equation {25} and into the boundary condition Eq. {16}. Altogether the energy of the falling mass is as follows:

$$E(r, v) = m_0 c^2 \cdot \frac{\sqrt{1-\frac{R_S}{r}}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \{26\}$$

2.7. Classical approximation of the exact energy

In order to compare the exact energy in Eq. {26} with the classical energy, the students realized that the ratios $\frac{R_S}{r}$ and $\frac{v^2}{c^2}$ are very small compared to one in a classical system. Accordingly, they expressed the Lorentz factor as a function of $\frac{v^2}{c^2}$ by the tangent. The tangent describes the linear order and is marked by a dot above the equality sign:

$$\gamma\left(\frac{v}{c}\right) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \doteq 1 + \frac{1}{2}\frac{v^2}{c^2} \qquad \{27\}$$

Similarly, the students derive the position factor in linear order:

$$\varepsilon\left(\frac{R_S}{r}\right) = \sqrt{1 - \frac{R_S}{r}} \doteq 1 - \frac{1}{2}\frac{R_S}{r} \qquad \{28\}$$

Thus, the exact energy is expressed in linear order as follows:

$$E(r, v) \doteq m_0 c^2 \cdot \left(1 + \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{2}\frac{R_S}{r}\right) \qquad \{29\}$$

We applied the distributive law and the abbreviation in Eq. {23}:

$$E(r, v) \doteq m_0 c^2 + \frac{1}{2} m_0 v^2 - \frac{G \cdot M \cdot m_0}{r} \qquad \{30\}$$

In this term, the students identified the classical kinetic energy

$$E_{kin} \doteq \frac{1}{2} m_0 v^2, \qquad \{31\}$$

The classical potential energy

$$E_{pot} \doteq -\frac{G \cdot M \cdot m_0}{r} \qquad \{32\}$$

And the relativistic energy of a mass m_0:

$$E_0 = m_0 c^2 \qquad \{33\}$$

2.8. Derivation of the Schwarzschild metric

The students derived the Schwarzschild metric in the form of the line element ds^2, see Eq. {8}. For it, they used the line element describing the effect of the velocity, see Eq. {17}. In free fall, the Lorentz factor is equal to the inverse position factor, according to the law of energy conservation in Eq. {17}. So the effect of the velocity can be transformed to the effect of the position:

$$ds^2 = -c^2 dt^2 \varepsilon^2 + \frac{dx^2}{\varepsilon^2} + dy^2 + dz^2 = 0 \quad \{34\}$$

This line element describes the falling mass at the radius r for the case that the velocity has been slowed down to zero. In principle, the above Eq. represents the Schwarzschild metric, see e. g. Schwarzschild (1916) or Hobson (2006). In the usual form, spherical polar coordinates are used:

$$dy^2 + dz^2 = r^2 d\theta^2 + r^2 \cdot \sin^2(\theta) \cdot d\phi \quad \{35\}$$

Additionally, usually, the position factor is replaced according to its definition:

$$ds^2 = -c^2 dt^2 \cdot \left(1 - \frac{R_S}{r}\right) + \frac{dr^2}{1-\frac{R_S}{r}} + r^2 d\theta^2 + r^2 \cdot \sin^2(\theta) \cdot d\phi \qquad \{36\}$$

2.9. Applications of the Schwarzschild metric

The students applied the Schwarzschild metric in order to explain the gravitational lens in Fig. 1. For details see Carmesin (2018a,b) or Burisch et al. (2022, p. 488). Moreover, they used that metric in order to explain the black hole, Burisch et al. (2022, p. 489). Additionally, the students explained autonomous cars with help of the derived general relativity, see Burisch et al. (2022, p. 489). Furthermore, the learners used the concepts of general relativity in order to derive results about the expansion of space since the Big Bang, see e. g. Burisch et al. (2022, p. 492-502) or Carmesin et al. (2020, pp. 296-302). Additionally, the students applied the concepts of general relativity to gravitational waves, see e. g. Burisch et al. (2022, p. 518-519) or Carmesin (2017).

3. Didactic analysis

The approach to relativity presented above is based on the following didactic steps:

3.1. **Didactic steps in special relativity**

In a first didactic step, the students discover the invariance of the velocity of light. For it, they use the light curve (Fig. 3) of the star W Ursae Mayoris (Fig. 2), obtained by students in the school observatory. Thereby, the cognitive conflict with the Galileo transformation in everyday life is elaborated. Hereby, the students realize that the invariance is a completely new and fundamental principle. The used learning material has the advantage, that students can obtain that result by observation on their own, in principle. Moreover, the used light emitted by a binary star can be extended to a thought experiment that provides the invariance of the velocity of light, see Carmesin (2022). There is no mathematics used here, so the technical learning barrier is very low. Furthermore, the students can obtain the hypothetic light curve corresponding to the basic Galileo transformation by a computer simulation, see Carmesin (2006).

In a second didactic step, the students derive the time dilation and the Lorentz factor. Mathematically, only the theorem of Pythagoras is used. So the technical learning barrier is very low. Hereby, the students realize that the usual assumption of everyday life and of Newton (section 1) of constant rate of increase of time is invalid in general. Usually, this is very inspiring to the students. This effect can be made very transparent with help of the twin paradox, see e. g. Burisch et al. (2022, p. 478-479) or Carmesin (2016).

In a third didactic step, the light-travel distance is used in order to derive the Lorentz contraction. Mathematically, only the third binomial formula is used. So the technical learning barrier is very low. The result is very inspiring, as it shows that space and time are combined to spacetime.

In the didactic step four, a light flash is used in order to develop the line element ds^2. Mathematically, only the theorem of Pythagoras is used. So the technical learning barrier is very low. The result is very propädeutic, as it is extended to a representation of the Schwarzschild metric in section 2.8.

In a fifth didactic step, the relativistic energy is derived from the time dilation, see Eq. {9}. Mathematically, this derivation requires an integration, see Burisch et al. (2022, pp. 480-483). Alternatively, the relativistic energy can be obtained from observation, see Carmesin et al. (2020 p. 49). Hereby, the mathematical and the conceptual learning barrier are both very low.

3.2. **Didactic steps in general relativity**

In the didactic step six, the concept of the investigation of acceleration in general relativity and the use of the acceleration sensor are elaborated. The developed ideas are very inspiring, and there is no mathematical learning barrier.

In the seventh didactic step, the equivalence principle is obtained by observation at the free fall tower. The students like that experiment very much, as they can experience and measure the state of free fall. The analysis of the data (Fig. 10) is very elucidating, and the concept of zero gravity at free fall is very surprising and inspiring to most students. Moreover, the learners realize that the equivalence principle is very powerful, as it combines gravity and motion is a precise and clear manner.

In didactic step eight, the conditions of the principle of energy conservation are analysed. This didactic step can be omitted, as most learners have no doubts about that concept.

In didactic step nine, the exact energy function is derived. Hereby, several derivatives have to be evaluated, including a test of an Ansatz by inserting into a derived differential equation. So the mathematics of differentiation is required. In comparison to the usually used mathematic of differential geometry, the proposed learning process has a very low learning barrier. The result is fully exact. Thus, this didactic step is regarded as highly efficient.

In a tenth didactic step, the classical energy is derived. For it, a linear approximation or the functional term of a tangent is elaborated. Thus, the mathematics of differentiation is required. In principle, that step could be omitted, as it is not used in the following. However, the students like that step very much, as it connects the new exact energy to the well-known classical energy. According to Hattie (2009), the learning efficiency is very high (1.48), as new and previous knowledge are connected in the field of science.

In didactic step eleven, the Schwarzschild metric is derived. The result is very inspiring, as it provides new insights in connects the Schwarzschild metric with the Minkowski metric, formally. Thus, this step has a very high learning efficiency according to Hattie (2009). Moreover, the achieved result is exact.

As this result is usually derived via the Einstein field equation in four-dimensional spacetime by using differential geometry, see e. g. Hobson (2006), the present learning process is especially efficient, as the exact result is obtained by a complete and one-dimensional analysis based on the mathematics of calculating derivatives only. This step provides the essential result, and it makes possible many applications, see section 2.9.

4. **Experiences with teaching**

I used this derivation of the Schwarzschild metric in several courses in the framework of a research club. Moreover I used it in several courses in general studies at the university.

In all groups, we used the results obtained by observation of the light curve and of the gravitational lensing, see Figs. (1-3). Only in few groups, we made the photos. The understanding of the principle of invariance of the velocity of light and the concept of gravitational lensing was good in all courses. Thus, the astronomical observation makes the learning process more intensive, but the observation by students is not necessary.

In all groups, we used the results obtained at the free fall tower. In most groups, we did not make the excursion. The understanding of the equivalence principle was good in all courses. Thus, the excursion makes the learning process more intensive, but it is not necessary.

All didactic steps have been achieved by all groups in a good manner, though the heights of the learning barriers are very different. This is achieved by adapting the amount of instruction to the respective learning barrier.

Altogether, all learning groups achieved the full insight to the Schwarzschild metric and to many of its applications. Thus, the present learning process provides full and exact participation of learners in an essential part of modern physics of spacetime. Thereby, the learners experience their own competence in an especially intensive manner, as they can elaborate and explain all steps (after an appropriate phase of instruction) on their own.

5. Discussion

Spacetime is very interesting to many students, as space and time are very fundamental concepts. Moreover, both concepts are used in everyday life.

Here, a learning process is presented that provides a high level of participation and competence. For it, all insights are achieved by the learners on their own, after an appropriate phase of instruction. Moreover, all results are derived from first principles in an exact manner. So, the new results are fully connected with previous knowledge, and a high learning efficiency is achieved, see Hattie (2009). Furthermore, the learners are enabled to participate in a discussion of the methods, results and applications at a high level. Additionally, learners obtain intensive experiences by making astronomical observations on their own and by experiencing free fall and the corresponding equivalence principle at the free fall tower. In this manner, learners experience their own competence and develop their self-esteem.

I tested the full learning process in several learning groups several times. I showed that the learning barriers are especially low, but the results are exact, very inspiring and elucidating as well as rich in content. The learning process is very robust and transparent. I provide a description of the learning process that can be used directly for teaching.

As the free fall tower and the Heidepark in general provide interesting and exciting processes in classical and in relativistic physics (the equivalence principle, for instance), students can explore and discover essential results in both: in Newton's mechanics (Carmesin 2014a,b,c) and in relativity.

6. Literature

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