Students Exactly Derive Quantization and its Universality

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Abstract

The development of renewable energy is essential for the future of our economy, society and climate. Hereby, photovoltaics has an enormous potential. Is it possible to improve its efficiency? In order to find an answer, students make a model experiment with light absorbed by LEDs. Thereby, they discover the Planck constant. With it, they propose multi junction photovoltaic cells providing a significantly increased efficiency. Moreover, students find the same Planck constant in a diffraction experiment with electrons. Apparently, the Planck constant is a universal constant. Why is the Planck constant universal? In order to find an answer, students use the concepts of classical light waves and relativity. Thereby, they derive the fact of quantization as well as the universality of the quantization constant. Here, I present a learning process, by which learners can achieve essential insights about photovoltaics and quantum physics in an exact manner. Thereby, students find essential questions directly by experiments. Using basic principles of physics, they achieve inspiring and exact results on their own, after an appropriate instruction. I present the learning process and a didactic analysis, so that you can directly use the concept for teaching. I tested the learning process in various learning groups, and I report about experiences.

1. Introduction

Present-day students show a clear tendency to become responsible for the energetic future of our planet. For instance, many students participate in Fridays for Future activities, see e. g. Zitterbarth (2021). Indeed, the use of renewable energies becomes urgent, in order to slow down and stop the anthropogenic climate change, see e. g. IPCC (2012, p. 170) or Arrhenius (1896). Thereby, photovoltaics has an enormous potential, see e. g. Federal Environment Agency (2010, p. 11). However, that potential is hardly developed nowadays, see e. g. Busch et al. (2022, p. 33). Accordingly, in advanced physics courses in classes 11-13, students like to find answers to the following question: Is it possible to improve the efficiency of photovoltaics?

In order to find answers to that question, the students expose an LED to light and measure the voltage generated by the LED, see e. g. Carmesin et al. (2020, pp 258-263). For instance, in Fig. 1 a blue LED is exposed to the light of a pocket lamp. Thereby, a green LED is connected and begins to shine, see pin 20. Moreover, two crocodile clips are attached to the pins of the blue LED and to a voltmeter. It shows that the blue LED generates a voltage of 2.137 V in this experiment. In a series of experiments, the students investigate the generated voltage U or energy $E_{El}. = U \cdot e$ of an electron that absorbed the light, as a function of the frequency f of the LED. Thereby, they discover that the energy is proportional to the frequency, with a factor h of the proportionality:

$$E_{El} = f \cdot h \qquad \{1\}$$

They measure the following factor:

$$h = 6.8 \cdot 10^{-34} \text{ Js} \qquad \{2\}$$

The learners are told that this factor is called Planck constant $h = 6.62607015 \cdot 10^{-34}$ Js.

1.1. Organization of the paper

I propose our learning process in section 2. In part 3, I provide a didactic analysis. Experiences with teaching are presented in part 4. We discuss our findings in section 5.

Fig.1: Students expose a blue LED to the light of a pocket lamp. The LED transforms the energy of the light into electric energy. With it, the connected green LED at pin 20 begins to shine. Moreover, the students attach two crocodile clips to the pins of the blue LED and to a Voltmeter. So they find out that the LED generates 2.137 V.

2. Learning process

The learning process is based on meaningful contexts, see e. g. Muckenfuß (1995).

2.1. Climate change and renewable energy

Initially, we use an actual message about the climate change, in order to remind that our climate is rapidly changed by the exhausted green-house gases, mainly carbon dioxide, see e. g. Carmesin et al. (2023a, pp 182-201). As a main solution, the students propose the use of renewable energies. Moreover, they propose and confirm that photovoltaics have the largest potential among the renewables, see e. g. Federal Environment Agency (2010, p. 11). Hereby, they realize that only a small percentage of the area in Germany is sufficient to provide our energy, see e. g. Carmesin et al. (2016, pp 34-37). Of course, it is always interesting to increase the efficiency of photovoltaics, as the whole world should change to renewable energies, see e. g. Federal Foreign Office (2016, p. 20). Accordingly, we ask the question: How can the efficiency of photovoltaics be improved.

2.2. Improving photovoltaics

In order to find an answer to this question, the students interpret Eq. {1}: That equation shows that the electron absorbs quanta of energy. Thereby, absorbed quanta depend of the frequency of light. Accordingly, light at a frequency f consists of quanta, called photon, with the following energy:

$$E_{photon} = f \cdot h \quad \{3\}$$

Of course, that interpretation is confirmed with additional experiments, see e. g. Carmesin et al. (2020).

With that interpretation, they realize that sunlight consists of several colours or wavelengths or frequencies. But an LED absorbs only a small interval (or energy-band) of frequencies or energies. Thus, the sunlight should be exposed to a stack of absorbing layers. Thus, they propose multi-junction cells. In fact, such cells achieve efficiencies of ca. 50 %. For details about this learning process, see Carmesin et al. (2012). After we realized that the energy of photons can be used efficiently, we asked: Can the momentum of a photon be used, for a repulsion, for instance.

2.3. Momentum of a photon

For it, the students use the relativistic energy

$$E = m \cdot c^2 \quad \{4\}$$

of electrons with a velocity v. The relation has been discovered earlier at a very low learning barrier, see e. g. Carmesin et al. (2020, p. 49):

$$\gamma^2 E^2 = E^2 - E^2 \cdot \frac{v^2}{c^2} = m_0^2 c^4 \quad \{5\}$$

Using Eq. {4} and the momentum $p = m \cdot v$, the learners derive:

$$E^2 = m^2 c^4 \cdot \frac{v^2}{c^2} + m_0^2 c^4 = p^2 \cdot c^2 + m_0^2 c^4 \quad \{6\}$$

As a photon has no rest mass m_0, the learners discover the energy momentum relation of photons:

$$E = p \cdot c \quad \{7\}$$

With it and Eq. {3}, the students discover the momentum of a photon:

$$h \cdot f = p \cdot c \quad \{8\}$$

or

$$p = \frac{h}{\lambda} \quad \{9\}$$

2.4. Momentum of an electron

In order to investigate the behaviour of electrons in an electron microscope, see e. g. Carmesin et al. (2020, pp 242-247) or in an LED, see e. g. Burisch et al. (2022, pp 452-457), the learners execute an experiment with diffraction of electrons, see Fig. 2. Hereby, they discover that the momentum of electrons also fulfils Eq. {9}. Apparently, the Planck constant is not related to a particular material or object. Thus, the Planck constant seems to be a universal constant. In a parallel research club, we ask: Why is the Planck constant universal?

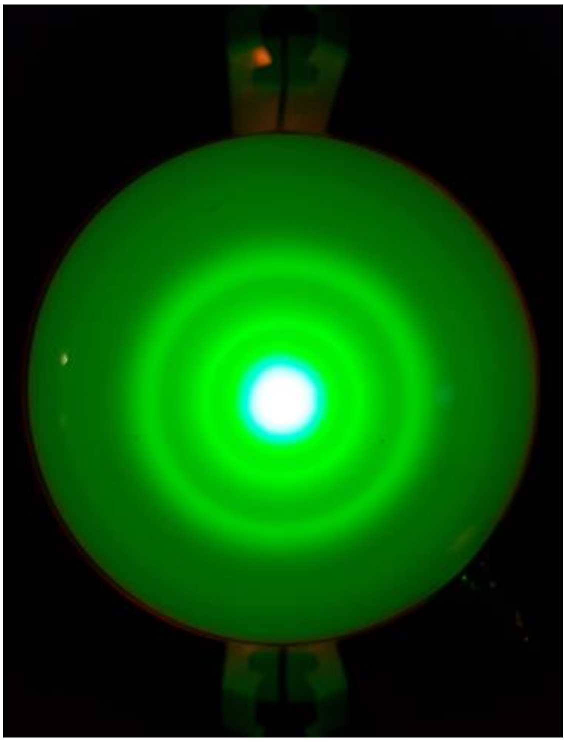

Fig.2: Students observe a diffraction pattern of electrons diffracted at polycrystalline graphite.

2.5. Quantization ratio

In order to investigate quantization, we reminded that we can already describe a monochromatic signal of light in terms of energy and momentum in Eq. {7} and in terms of the frequency and wavelength in Eqs. {1} and {9}. In order to combine both descriptions in a robust manner, we solved the relations for a common universal constant, the velocity of light:

$$\frac{E}{p} = c = \frac{f}{\lambda} = \frac{\omega}{k} \quad \{10\}$$

Hereby, k is the wavenumber, $\omega = 2\pi \cdot f$ is the circular frequency or angular velocity, and we use the absolute value p of the momentum. As the circular frequency is not zero, we can divide by ω. Additionally, we multiply by the momentum. Thus, the students use Eq. {10} in order to derive the following ratio:

$$\frac{E}{\omega} = \frac{p}{k} = K(\omega) \quad \{11\}$$

Hereby, the ratio $K(\omega)$ provides the relation between energy and frequency or circular frequency. Thus, that ratio is similar to the quantization constant in Eq. {1}. Accordingly, we call the ratio $K(\omega)$ quantization ratio. In order investigate that similarity, we considered a possible minimal portion $E_{min,\omega}$ of energy of a monochromatic signal with circular frequency ω. As we did not use the amount of energy in our derivation, Eq. {11} holds for the minimal energy as well:

$$\frac{E_{min,\omega}}{\omega} = \frac{p_{min,\omega}}{k} = K(\omega) \quad \{12\}$$

Thus, the quantization ratio describes possible minimal portions of energy, but we do not yet know, whether that ratio is the same for each circular frequency ω. Thus, we asked: Is the quantization ratio $K(\omega)$ independent of ω? In order to find an answer, we investigated the gravitational redshift, see e. g. Pound (1960) or Burish et al. (2022, p. 487) at a free fall tower.

2.6. Objects at free fall

In order to derive the universality of quantization, the students of the research club remind that a physical object such as a photon can be investigated at free fall. For the case of a probe mass m_0 at free fall, the energy function as follows, see Carmesin (2023b):

$$E(r,v) = m_0 c^2 \cdot \sqrt{1 - \frac{R_S}{r}} / \sqrt{1 - \frac{v^2}{c^2}} \quad \{13\}$$

Hereby, v is the velocity, and r is the distance from the probe mass to the field generating mass M. Moreover, the factor $\sqrt{1 - \frac{R_S}{Rr}}$ is a universal position factor $\varepsilon_E(r)$, see Carmesin (2023b):

$$E(r,v) = m_0 c^2 \cdot \varepsilon_E(r) / \sqrt{1 - \frac{v^2}{c^2}} \text{ with} \quad \{14\}$$

$$\varepsilon_E(r) = \sqrt{1 - \frac{R_S}{r}} \quad \{15\}$$

The students realized: In order to apply Eq. {13} to a photon, the relation must be generalized to an object with zero rest mass. For it, we generalized the factors in Eq. {13} according to their energetic content.

2.7. Available energy

When the mass m_0 is at free fall, then the potential energy is transformed to kinetic energy. At a radial coordinate r, the rest energy multiplied by the Lorentz factor is available for a transformation into another energy, we call that factor available energy:

$$E_{av}(r,v) = m_0 c^2 / \sqrt{1 - \frac{v^2}{c^2}} = \frac{E(r,v)}{\varepsilon_E(r)} = \frac{m_0 c^2}{\varepsilon_E(r)} \quad \{16\}$$

The students obtain the relations in Eq. {16} from Eqs. {13-15} and from the principle of energy conservation. In the case of a general object, a signal of monochromatic light, for instance, the energy $m_0 c^2$ in the limit r to infinity is replaced by E_∞:

$$E_{av}(r) = \frac{E_\infty}{\varepsilon_E(r)} \quad \{17\}$$

2.8. Light signal at free fall

In order to show that the quantization ratio $K(\omega)$ does not depend on ω, we consider to arbitrary circular frequencies ω_1 and ω_2, and we derive the relation of the corresponding quantization ratios $K(\omega_1)$ and $K(\omega_2)$.

As the light signal can be at each radius r, we analyse the signal with the circular frequencies ω_1 at the limit r to infinity. Hereby, we mark a physical quantity q in that limit by q_∞:

$$\omega_1 = \omega_\infty \quad \{18\}$$

In the Schwarzschild metric, the gravitational time dilation can be applied to increments dt and to periodic times T as follows, see Carmesin (2023b) or Burisch et al. (2022, pp 484-489):

$$\varepsilon_E(r) = \frac{dt(r)}{dt_\infty} = \frac{T(r)}{T_\infty} = \frac{\omega_\infty}{\omega(r)} = \frac{\omega_\infty}{\omega_2} = \frac{\omega_1}{\omega_2} \quad \{19\}$$

With it, the students derive energy and the available (or measurable) energy of the light signal during free fall, see Eqs. {12} and {17}:

$$E_\infty = K_\infty \omega_\infty \quad \{20\}$$

$$K(\omega_2)\omega_2 = E_{av}(r) = \frac{E_\infty}{\varepsilon_E(r)} = \frac{K_\infty \omega_\infty}{\frac{\omega_\infty}{\omega_2}} = K_\infty \omega_2 \quad \{21\}$$

The learners divide the above Eq. by ω_2:

$$K(\omega_2) = K_\infty = K(\omega_1) \quad \{22\}$$

With it, the students realize that the quantization ration does not depend on the circular frequency. Using the basic relations in Eq. {10}, the learners argue that the quantization constant does not depend on the wavelength or the wave number k or the momentum $p_{min,\omega}$. As there are no other possible dependencies in the definition of the quantization ratio in Eq. {12}, the quantization ratio is a universal quantity. Its value has been measured, see section 1.

3. Didactic analysis

The approach to quantization presented above is based on the following didactic steps:

3.1. Didactic Steps in developed quantum physics

In a first didactic step, the students realize that the anthropogenic climate change is an urgent global problem of mankind. By investigating technical facts, they identify renewable energy as a possible solution to the problem. Hereby, photovoltaic systems have the largest potential. This step is very motivating, and there is no high learning barrier.

In a second didactic step, the students realize that the implementation of renewable energy is a global problem, and for it, an increase of efficiency can be especially helpful, since some societies cannot afford large amounts of money for renewable energy. This step broadens the mind of the students, as the German or European perspective are insufficient, and as expensive solutions are not appropriate, as a consequence. This step has no high learning barrier.

In a third didactic step, the learners investigate photovoltaics with a model experiment with LEDs. Hereby, they discover the proportionality of electric energy and frequency of the LED, and they determine the corresponding constant, the Planck constant. Moreover, they interpret their finding, whereby they achieve the concept of quantization of light and of the photon. The learning material of the LED is very efficient:

- It is an everyday life device.
- It is available for many experiments in the learning group
- It provides relatively precise results, if LEDs with sufficiently high power are used.
- It is not dangerous, as the LEDs do not emit light, or they emit light at a very low intensity, see Fig. 1.
- It provides the Planck constant.
- Its interpretation provides the concept of quantization of light.

The learning barrier is intermediate in the measurements and in the evaluation. The barrier is large in the interpretation. Thus, additional experiments confirm the interpretation, see e. g. Carmesin (2020, pp 254-263).

In the didactic step four, the students propose multijunction photovoltaic cells consisting of several layers absorbing light at different energies. With it, the efficiency of photovoltaics is increased significantly, see Carmesin et al. (2012).

In a fifth didactic step, the learners analyse the momentum of photons, see Eq. {9}. Hereby, the mathematical learning barrier is low, as few equivalence transformations of known equations are used only. However, the learning barrier of the productive combination of known relations in relativity and quantum physics is high. Accordingly, the learning process should be planned and discussed with the students.

In the didactic step six, the students investigate electron microscopes with help of a model experiment with electron diffraction. Hereby, usually, only one experiment is available, so that the experiment is executed in the plenum, not in groups. The result is very inspiring, as matter waves are discovered. Thereby, the same constant of quantization and the same relation of momentum and wavelength are discovered. Accordingly, an essential learning barrier is the limited knowledge about the nature of the wave function. This is partially overcome within the course with help of the stochastic interpretation, the uncertainty relation and entanglement, see e. g. Carmesin (2020, pp 264-289). More fundamental results about the nature of the wave function are provided in Carmesin (2022a, 2022b, 2022c, 2023c). Hereby, the following question arises: Why is the quantization ratio the same in different fields of nature or physics? The fact of this universality of the quantization constant has to be obtained according to the official curriculum, see Bresser et al. (2022, pp 40-42).

3.2. Didactic steps in universal quantization

The above question is treated in a research club by the following didactic steps:

In the seventh didactic step, the learners derive the quantization constant for the case of a monochromatic light signal. Hereby, they obtain the exact result. For it, they directly combine their knowledge about a monochromatic light signal in Eq. {10}. They execute a simple equivalent transformation in order to derive the exact quantization in Eq. {11}. It is conceptually simple, to use that ratio for the most interesting case of a possible minimal energy of the signal in Eq. {12}. By comparing the result with the Planck constant in Eq. {1}, the learners ask the following question: Why does the quantization constant not depend on the circular frequency? In this step, all technical, learning barriers are very low. Thus, there is a lot of time for the discussion of the results. The students like such discussions, and the learners are fully enabled to execute discussions at a high level, as they have a full overview of the transparent and complete derivations.

In didactic step eight, in order to find an answer to the above question, the learners analyse the light signal and its circular frequency at a free fall tower. For it, the concept of the available energy is introduced, see Eq. {17}. That concept is simple, it can be derived from the well-known behaviour of a mass at a free fall tower (Eq. {13}), it is transparent, and it is confirmed by the observation of the gravitational redshift, for instance. Thus, this didactic step has a low learning barrier.

In didactic step nine, the learners compare the energy and available energy of an object at free fall. With it, they derive the universality of the quantization constant, see section 2.8. Hereby, the mathematical barrier consists of simple equivalence transformations of equations only, so it is low. The derivation is guided by the idea of comparison. In order to lower the learning barrier, a plan for the comparison is developed in the plenum. In this manner, an appropriate learning barrier is achieved.

4. Experiences with teaching

I report about experiences in classes 11-13 in physics, in classes 8-13 in a research club and in general studies courses at the university.

4.1. Experiences in developed quantum physics

I used the development of the quantization in the didactic steps 1-6 in several courses in physics in classes 11-13 at the high school. The used contexts are very motivating. In addition, the contexts decrease the cognitive learning barrier, as the students can use competences about everyday life. The students achieved good results in the written examinations within the semesters and in the final examinations.

4.2. Experiences in derived universal quantization

I used the derivation of the universal quantization in the didactic steps 7-9 in a research club with students of classes 8-13. Additionally, I used these didactic steps 7-9 in several courses in general studies at the university.

The results of all three didactic steps have been achieved by all learning groups in a good manner. This good performance was supported by the fact that all three didactic steps have a low mathematical learning barrier. Moreover, the learners benefited from the fact that all derivations provide exact results only. In this manner, no cognitive load was caused by any approximation. Furthermore the students had the advantage that no hypothetic result was used at all. Consequently, no cognitive load is caused by any hypothetic consideration, that always should be treated in an especially careful manner. As unnecessary cognitive load is avoided, the students can focus their full attention to the essential physical arguments and achieved insights. Thereby, the remaining cognitive learning barriers are minimized. These remaining learning barriers are treated with an early phase of common planning, a phase of derivation or problem solving with an appropriate amount of instruction and a phase of common discussion of the results.

4.3. Experiences in all didactic steps 1-9

The proposed and tested learning process provides full participation of learners in experimental foundations, essential applications and the exact derivation of universal quantization. Thereby, the learners experience their own competence in an especially intensive manner, as they can explain all steps (after appropriate phases of planning, instruction, problem solving and discussion) on their own.

5. Discussion

Climate change, renewable energies, quantum physics and exact derivations of universal quantization are very interesting to many students. Thereby, quanta are essential for photovoltaics, and many objects consist of quanta. For instance, light, electrons, atoms and molecules consist of quanta.

Here, a learning process is presented that provides a high relevance to climate change and renewable energies and a level of participation and competence. For it, all insights are achieved by the learners on their own, after appropriate phases of planning, instruction and problem solving, as well as discussion. In this manner, learners experience their own competence and develop their self-esteem.

Moreover, all results are derived from clear experiments or from first principles in an exact manner. So, all new results are fully connected with contexts and with previous knowledge, so that a high learning efficiency is achieved, see Hattie (2009).

I tested the full learning process in several learning groups ranging from classes 8-13 and to general studies courses at the university. I showed that the learning barriers and cognitive loads are especially low for several reasons:

- Low mathematical barriers
- No hypothetic results
- No approximation

Altogether, the learners achieve various essential insights in a context based and founded manner:

- Climate change is an urgent global problem of mankind.
- Renewable energies can solve this problem.
- Photovoltaics provide the largest potential in renewable energies.
- The efficiency of photovoltaics can be improved with help of quanta.
- The constant of quantization is universal.
- The fact and universality of quantization can be derived from classical light waves, special relativity of energy and gravitational time dilation in an exact manner.

6. Literature

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