How Excitations of the Vacuum form Mass

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Kurzfassung

Materie ist ein zentrales Konzept. Sogar fundamentale Wechselwirkungen können durch Elementarteilchen mit Masse vermittelt werden. Das stellte die Physik zunächst vor ein Problem, denn die üblichen angewendeten Prinzipien der kleinsten Wirkung und der Eichinvarianz alleine sagen masselose Bosonen der Wechselwirkung vorher. Als Lösung wurde ein Phasenübergang vorgeschlagen, bei dem aus dem Vakuum Masse entsteht, das ist der Higgs-Mechanismus. Allerdings lässt der Mechanismus zentrale Fragen unbeantwortet: Wie genau entsteht diese Masse? Welches Massenspektrum tritt auf? Hier wird meine neue Theorie des Vakuums vorgestellt, die alle Parameter des Standardmodells der Kosmologie liefert, und die auch die beiden obigen Fragen beantwortet. In diesem Aufsatz analysiere ich die didaktische Perspektive des Themas, einschließlich Higgs-Mechanismus und Dynamik sowie Anregungsspektrum des Vakuums. Die Thematik habe ich in zwei Lerngruppen erprobt, in einer Jugend forscht Arbeitsgruppe für die Klassenstufen 8 bis 13 und in einem General Studies Kurs an der Universität Bremen. Ich berichte ich über Erfahrungen mit der Thematik in den beiden Lerngruppen.

Abstract

Matter is an essential concept. Indeed, also fundamental interactions can be transmitted by elementary particles with mass. This fact had become a problem, as the usual principles of least action and gauge invariance predicted particles of interaction without mass. That problem has been solved by the proposed Higgs mechanism: vacuum exhibits a phase transition that forms mass. However, that mechanism does not provide answers for essential questions: By what mechanism can vacuum generate mass? What spectrum does the vacuum provide in the process of generating mass? Here, my theory of vacuum is presented. That theory provides all parameters of the standard model of cosmology, and that theory provides answers to the above two questions. In this paper, I analyse the didactical perspective of the topic, including the Higgs mechanism as well as the dynamics and spectrum of vacuum. I tested that topic in two learning groups: a research club for classes 8 to 13 and a general studies course at the university Bremen. I report about the experience with the use of the topic in the two learning groups.

1. Introduction

Students are highly interested in astronomy and astrophysics, including space science (Elster 2010, Jenkins 2006, Pössel 2015). An especially interesting feature of space is its expansion since the Big Bang, and how that expansion is caused by the formation of vacuum. A particularly interesting feature of vacuum is its ability to form mass. Such a process takes place in the formation of the Higgs boson, for instance. As mass and matter are ubiquitous in everyday life, that process is essential for understanding the characteristics of nature, which is another very interesting topic for students (Elster 2010). The transformation from space to matter is caused by a phase transition.

Students know phase transitions from everyday life: They know about condensation of water providing rain or fog, for instance. They know the inverse transition of evaporation. Additionally, they know about the melting of ice and the inverse transition: freezing. Moreover, they know how molecules are organized at these transitions. Thus, the students know about the general concept of phase transitions, but they do not know how vacuum can provide such a transition. So, the students experience a substantial scientific curiosity, when they are posed to the question: How does matter form from vacuum in the process of a phase transition?

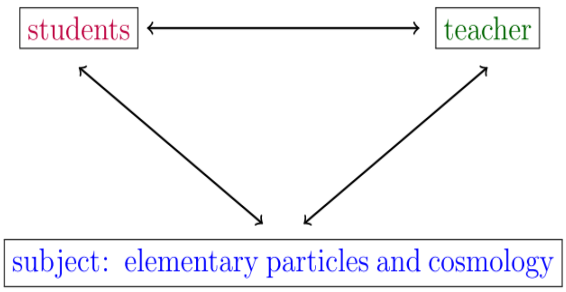

Fig. 1: Didactical triangle.

1.1. Didactical perspective

The didactical perspective can be analysed in the framework of the didactical triangle, see Franke and Gruschka (1996) or Fig. 1. That triangle represents the relation between the students, the teacher and the subject. Hereby, the relation between the students and the subject is characterized by a high interest of the students, including the motivating aspect of scientific curiosity. From the perspective of the students, the idea of a phase transition forming matter from vacuum poses several questions, see Fig. 2: What structure of vacuum causes a transition? Why and how is vacuum reorganized at the transition? What structure of matter is provided by the transition?

In the perspective of the history of science, the phase transition from vacuum to matter poses similar questions: The structure of matter has been a mystery for a long time. More than 2000 years ago, Leukippos and Democritos proposed hypothetical particles (named 'atomos', see Tsaparlis 2001) as the smallest and indivisible building blocks of matter. Dalton (1808) provided a scientific basis for atoms. In present-day physics, many elementary particles have been discovered at length scales down to 10^{-20} m and at energy scales up to TeV, see e. g. Zyla (2020). These elementary particles are usually described by the standard model of elementary particles, SMEP, see Weinberg (1996). In that model, the formation of mass is described by a phase transition, by which mass forms from vacuum (Higgs 1964). However, in the SMEP, two essential questions are not answered:

What mechanism of vacuum forms the mass? What spectrum of vacuum is underlying the formation of mass?

The questions are summarized in the didactical triangle in Fig. 2. These questions are answered in this paper. This is achieved by my new theory of vacuum (Carmesin 2021a, Fig. 1). With it, quantum physics, QP, (Carmesin 2022a) and general relativity, GR, (Carmesin 2022b) have been derived. Moreover, essential properties of vacuum (Carmesin 2020) and of particles, that are not predicted by the SMEP, have been derived (Carmesin 2021b-e, 2022b). Thus, the theory of vacuum, QP, GR and the SMEP are related as shown in Fig. 4.

As an advance organizer, I summarize the dynamics of vacuum: Vacuum represents a concept of physics, whereas concepts of space are used in mathematics and in physics. Basically, the dynamics of vacuum have been derived from fundamental physical principles including quantum physics, see e. g. Carmesin (2017, 2018a, b, 2019a, 2021a-f). Moreover, quantum physics has been derived from gravity and special relativity only (Carmesin 2022a).

In particular, vacuum dynamics provides various results: Basically, the dynamics of vacuum describes three-dimensional vacuum, and more generally, the dynamics of vacuum includes higher dimensional vacuum, see e. g. Carmesin (2021a, f). Vacuum dynamics have been analysed at a semiclassical level and at a quantum physical level, see e. g. Carmesin (2021a, f). Basically, vacuum dynamics describes the formation of space, moreover, it describes the formation of matter as well as the formation of elementary charges, couplings and of fundamental interactions, see Carmesin (2021a-f, 2022b, c). Basically, the dynamics of vacuum describes the rate of expansion of space since the Big Bang, including the Hubble constant H_0. Furthermore, vacuum dynamics describes the Hubble tension and the era of inflation (Carmesin 2021a-c). Vacuum dynamics describes the propagation of the gravitational interaction, see Carmesin (2021a). Basically, vacuum dynamics describe the formation and propagation of vacuum at a far distance of a possible black hole with a Schwarzschild radius R_S, $R_S/R>>1$. Moreover, the dynamics of vacuum describes the formation and propagation of vacuum in the vicinity of a black hole Carmesin (2022a, Eq. 3.250-3.252).

The concept of the Higgs-mechanism including the Higss-field has been used in classes 11 and 12, see Becker and Hopf (2021). Thereby, the students understood the basic idea and applied it in new examples by transfer. In that study, the Higgs-field has been used as a basic physical entity. Here, that field is explained by the dynamics of vacuum.

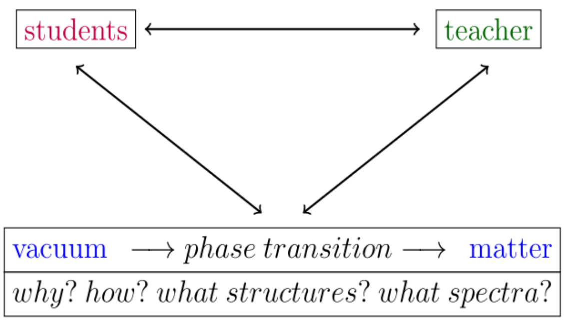

Fig. 2: Didactical triangle: scientific curiosity.

1.2. Didactical perspective of vacuum

The scientific question about the phase transition can be treated in an analytic and productive manner in two steps. I presented that concept to learners ranging from class 8 to 13 in a research club (see Carmesin, 2021f, Lieber and Carmesin 2021, Carmesin and Carmesin 2020) and to students of a general studies course at the university Bremen. In both groups, the students were able to describe the steps of the respective derivations and to discuss the consequences. Moreover, some advanced students derived some results on their own. Some of these presented their results at Jugend forscht competitions and won prizes. In particular, the students of both groups achieved the following competences:

Firstly, the missing mass in the electroweak interaction is analysed, and the Higgs mechanism provides a solution, see Fig. 3. Moreover, the Higgs mechanism provides the concept of an expectation value (a so-called vacuum expectation value, vev) of the energy of a portion of vacuum. However, the values, the portion of vacuum and the underlying mechanisms are not modelled or clarified.

Secondly, the dynamics of the vacuum is analysed. These dynamics of vacuum provides several didactical perspectives, see Fig. 3.

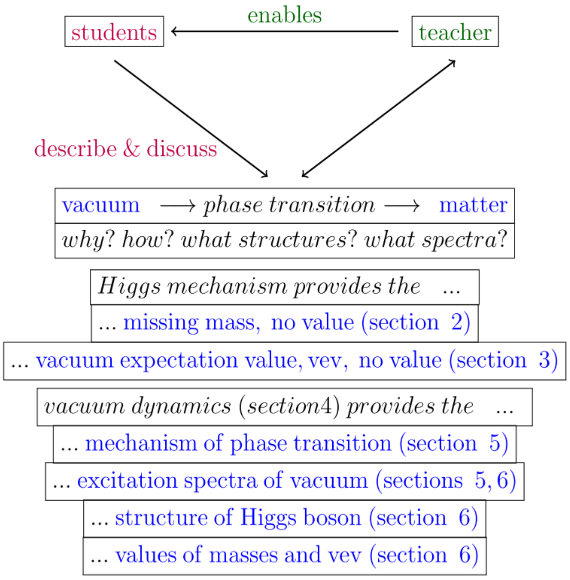

Fig. 3: Didactical triangle illustrates students describing and discussing the formation of mass from vacuum at two levels: Higgs mechanism and dynamics of the vacuum.

The students described and discussed the steps of the derivation of the dynamics of vacuum (section 4). Moreover, the students described and discussed, how the dynamics of vacuum provide dimensional phase transitions at high density (section 5). Hereby, the students realized the following items: Firstly, the dynamics of vacuum have been tested. Secondly, dimensional phase transitions are a natural consequence of the dynamics of vacuum. Thirdly, higher dimensional physics have been observed in experiments (Lohse et al. 2018, Zilberberg et al. 2018). Fourthly, dimensional phase transitions naturally provide the era of cosmic inflation. Fifthly, the critical densities have been derived and evaluated. Sixthly, these critical densities explain the everyday life fact of three-dimensional space (D = 3 is non-trivial and requires an explanation). On the other hand, the students pointed out that the concept of a dimensional phase transition is surprising.

Furthermore, the students described and discussed the following: the excitation spectra of the vacuum (sections 5, 6), the minimal structure of the Higgs boson and the derived mass, which is in accordance with observation (section 6), the structure of the vev and the derived masses and energies, according to observations (section 6). Altogether, didactical perspectives inherent to vacuum have been used at two levels: the level of the Higgs mechanism and the level of vacuum dynamics.

2. Missing mass in electroweak theory

In this section, we shortly summarize the unification of electromagnetic and weak interactions to the electroweak interaction, EWI (Weinberg 1996). For instance, that unified interaction describes the β-decay. Hereby, an up-quark decays into a down-quark, an electron and an electronic antineutrino:

$$u \rightarrow d + e^- + \bar{\nu}_e \qquad \{1\}$$

The corresponding Lagrangian or energy function or Hamiltonian can be derived from quantum physics, which can be described by the principle of least action or extremal action, PLA (see Erb 1992 or Grebe-Ellis 2006 for didactical perspectives and applications of the PLA), in quantum field theories (Carmesin 2022a, Carmesin 2022b, Feynman 1985, Weinberg 1996). In particular, the interaction can be derived from the principle of gauge invariance, PGI (Carmesin 2022a, Carmesin 2022b, Weinberg 1996). Thereby, the electroweak interaction is transmitted by the photon, the Z-boson and the W-bosons. Hereby, these bosons have zero mass, according to the PLA and the PGI. However, the observed mass of the Z-boson is 91,1876 GeV and the observed mass of a W-boson is 80,38 GeV (Zyla 2020):

$$m_Z = 91{,}1876\ GeV;\ m_W = 80{,}38\ GeV \qquad \{2\}$$

Thus, there is a huge missing mass in the PLA and PGI.

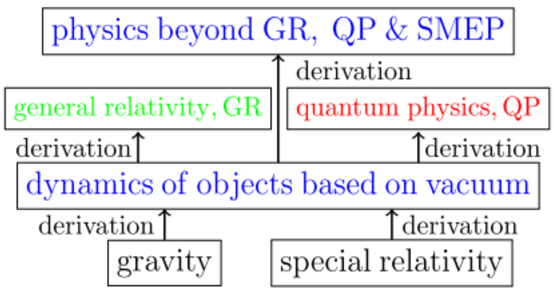

Fig. 4: My theory of vacuum can be used to derive quantum physics and general relativity as well as physics beyond QP, GR and the SMEP.

3. Mass via phase transition of vacuum: Higgs mechanism

In this section, we shortly summarize the Higgs mechanism (see Higgs 1964 or Weinberg 1996). The theory is based on the relativistic relation of energy E and moment p (Einstein 1905):

$$E^2 = p^2c^2 + m_0^2c^4 \qquad \{3\}$$

In the theory of the EWI, the term p^2c^2 represents the kinetic energy T and the term $-m_0^2c^4$ represents the potential energy E_{pot} or the potential V. Moreover, the square of the mass is multiplied by the square of a scalar field Φ^2. Note that I derived such a scalar field from the theory of vacuum and from the wave function (Carmesin 2022a, b). Additionally, we use

Planck units, so that the velocity of light is one. Accordingly, the potential is as follows:

$V(\Phi) = -m_0^2 \cdot \Phi^2$ {4}

Higgs proposed an additional term $\lambda \cdot \Phi^4$, with a so far unknown parameter λ:

$V(\Phi) = -m_0^2 \cdot \Phi^2 + \lambda \cdot \Phi^4$ {5}

Thereby, Higgs proposed a complex or a two-dimensional real field, so that a component Φ_1 is described as follows:

$\Phi_1 = \frac{\Phi}{\sqrt{2}} = \frac{v}{\sqrt{2}}$ {6}

Thus, Higgs obtained a usual potential describing phase transitions (Landau 1937): If m_0 is positive, then the minimum of the potential occurs at the following value of the scalar field:

$\Phi_{1,\mathrm{opt}} = \frac{m_0}{\sqrt{4\lambda}} = \frac{v_{opt}}{\sqrt{2}} = \frac{\Phi_{opt}}{\sqrt{2}}$ {7}

Hereby, and usually, Φ is denoted by a variable v, whereby that variable is an abbreviation of vacuum. According to the usual interpretation, that the vacuum provides the mass as a result of a phase transition. The variable v marks the vacuum expectation value. For details, see Zyla (2020) or Carmesin (2022b). Observations provide the vev, see Zyla (2020):

$\mathrm{v_{opt}} = 246.1965\ GeV \pm 0.6\ ppm = vev$ {8}

Altogether, the Higgs mechanism provides the missing mass, see above section, but it does not provide a theory of the vacuum. After I derived a theory of the vacuum (Carmesin 2017, Carmesin 2018a, b, c, Sprenger and Carmesin 2018, Carmesin 2019a, b, Carmesin 2021a), which derives and explains all parameters of the standard model of cosmology (Carmesin 2021b), and which derives and explains the Hubble tension or the local value of the Hubble constant (Carmesin 2022c, Riess 2022), and which derives and explains quantum physics (Carmesin 2022a), I analysed, whether that theory of vacuum can derive the masses in Eq. {2} and the vev in Eq. {8}. In fact, the theory of vacuum provides these results, in precise accordance with observation, and without executing any fit (Carmesin 2022b).

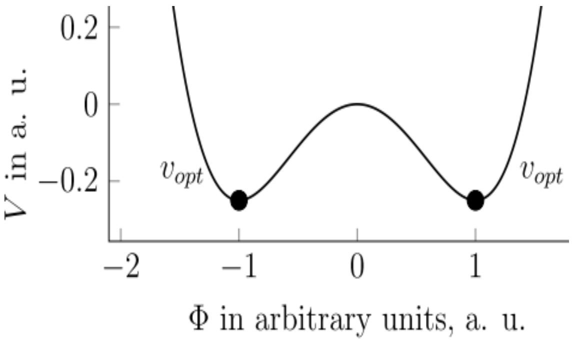

Fig. 5: A potential $V(\Phi)$ describes the formation of mass by a phase transition according to the Higgs mechanism. At the phase transition, the field Φ changes from 0 to 1 at the minimum Φ = v_{opt}. That nonzero value of Φ represents the rest mass m_0 forming at the phase transition.

4. Formation of vacuum

In this section, I summarize essential results of my theory of vacuum. A mass M with a Schwarzschild radius $R_S = 2GM/c^2$ forms vacuum at a coordinate distance R from M. Hereby, $G = 6.67430(15) \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$ is the gravitational constant, and $c = 299\,792\,458 \frac{m}{s}$ is the velocity of light (Zyla 2020). In particular, I derived the corresponding differential equation, DEQ. I derived this result directly from gravity and relativity (Carmesin 2022a, section 3.9). A shorter derivation is based directly on the Schwarzschild metric (Fig. 6). That derivation is presented in Carmesin (2022d).

A mass M with a Schwarzschild radius $R_S = 2GM/c^2$ causes curvature of spacetime (Schwarzschild 1916 or e. g. Carmesin 1996). The radial component of that curvature represents the elongation of a radial coordinate difference dR to a physical length dL as follows:

$dL = dR \cdot \left(1 - \frac{R_S}{R}\right)^{-\frac{1}{2}}$ {9}

Next, we analyse the elongated length dL in the limit of limit R_S/R to zero, we name that limit the *far distance limit*. For it, we apply the first order Taylor approximation:

$dL \doteq dR \cdot \left(1 + \frac{R_S}{2R}\right)$ {10}

Thus, the observable increase of length is as follows:

$\delta R = dR \cdot \frac{R_S}{2R}$ {11}

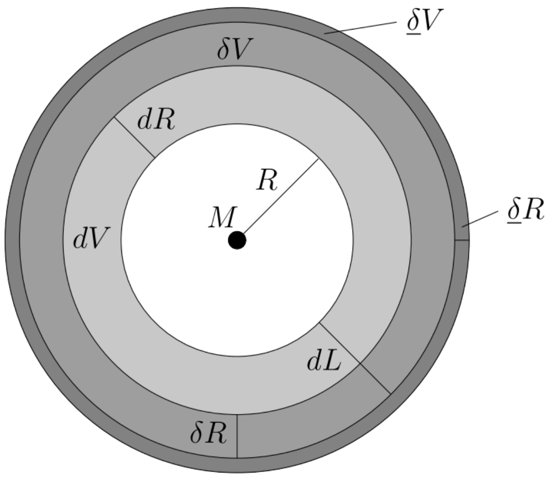

Fig. 6: Formation of vacuum caused by a mass M.

Hence, there occurs an observable increase of volume δV as follows:

$\delta V = 4\pi R^2 \delta R = 2\pi R_S R \cdot dR$ {12}

Moreover, that additional vacuum δV propagates at the velocity of light, and it flows radially away from the mass M. We ask: Where is the origin of that vacuum? In order to derive an answer, we analyse, whether the observable vacuum δV increases as a function of R. For it, we analyse a time-interval $\underline{\delta}t$, in which the vacuum δV propagates a radial difference

$\underline{\delta}R = c \cdot \underline{\delta}t$ (Fig. 6). We analyse the difference $\underline{\delta}V$ of the vacuum δV, that occurs during that propagation:

$$\underline{\delta}V = \frac{GM}{R^2} \cdot dV \cdot \frac{\delta R}{c^2} = \frac{GM}{R^2} \cdot dV \cdot \frac{\delta t}{c} \quad \{13\}$$

We identify the first fraction with the gravitational field at R:

$$G^*(R) = \frac{GM}{R^2} \quad \{14\}$$

Thus, the new volume $\underline{\delta}V$ that formed from the present volume dV during a time $\underline{\delta}t$ is as follows:

$$\underline{\delta}V = \frac{G^*}{c} \cdot dV \cdot \underline{\delta}t \quad \{15\}$$

We name the ratio of the new vacuum $\underline{\delta}V$ formed from the present vacuum dV by the *relative formation* ε:

$$\varepsilon = \frac{\underline{\delta}V}{dV} \quad \{16\}$$

During that process, the present volume dV is constant. Thus, within the Leibniz calculus, the time derivative of the relative formation ε is as follows:

$$\dot{\varepsilon} = \frac{\underline{\delta}V}{dV \cdot \underline{\delta}t} \quad \{17\}$$

This time derivative represents the (relative) *rate of formation of vacuum*. We solve the above equations for the rate of formation of vacuum:

$$\dot{\varepsilon} = \frac{G^*}{c} \quad \{18\}$$

This DEQ describes the formation of vacuum caused by a mass M. Many properties of that formed vacuum have been analysed in detail in Carmesin (2021a).

More generally, I analysed that formation of vacuum in dimensions $D \geq 3$ of space. For the case of the far distance limit, the result is as follows (Carmesin 2022e):

$$\dot{\tilde{\varepsilon}} = (D-2) \cdot \tilde{G}^* \quad \{19\}$$

Hereby, variables with a tilde are represented in Planck units, see e. g. Carmesin (2021a). The gravitational field in $D \geq 3$ dimensional space is as follows:

$$\tilde{G}^* = \frac{\tilde{M}}{\tilde{R}^{D-1}} \quad \{20\}$$

5. Phase transitions of vacuum derived

In this section, we summarize results about phase transitions that take place at high density $\tilde{\rho}$ in the vicinity of the Planck density $\tilde{\rho} \approx \tilde{\rho}_P$ in the vacuum described by Eq. {20}.

When space at a dimension $D \geq 3$ makes a transition to a dimension $D+1$, then the D-dimensional space can be regarded as a hypersheet in $D+1$-dimensional space. Hereby, the thickness of that hypersheet is so small, that it cannot be observed. Thus, the thickness is equal to the Planck length L_P, as a smaller thickness cannot be observed and is unphysical, and since a larger thickness could be observed, so that there would already be in $D+1$-dimensional space. We describe the transition by the formation of connections between neighbouring hypersheets. The most likely and thus relevant connections have the smallest possible size. Hence, these connections have the extension L_P in all directions. So, they can be modelled by cubes with length L_P. It has been shown that for each dimension $D \geq 3$, the described connections form spontaneously, as soon as the density exceeds a critical density as follows (Carmesin 2021a, p. 83):

$$\tilde{\rho}_{cr,conn.} = 0.018 \cdot \tilde{\rho}_P \quad \{21\}$$

5.1. Dimensional horizon

In this section, we summarize the results about the dimensional horizon.

The space within the present-day light horizon experienced a time evolution. It can be described by the time evolution of the light horizon $R_{lh}(t)$ or by the time evolution of the Hubble radius $R_H(t)$. If the dimension is increased, then the light horizon $R_{lh}(t)$ is decreased. When the light horizon $R_{lh}(t)$ is as long as one Planck length, then the dimension D of the space within the present-day light horizon cannot be increased further. That dimension is called dimensional horizon D_{hori} (see e. g. Carmesin 2017, 2019a, 2021a):

$$\text{If} \quad R_{lh} \approx L_P, \text{ then } D = D_{hori} \quad \{22\}$$

In our universe, the dimensional horizon is approximately 301:

$$D_{hori} \approx 301 \quad \{23\}$$

5.2. Distance enlargement factor

In this section, we summarize the results about the distance enlargement, that takes place at a dimensional phase transition.

If the space has a dimensional horizon D_{hori}, and if its dimension changes from $D+s$ to D, then the space is enlarged by the following dimensional distance enlargement factor (see e. g. 2019a, 2021a, 2022c):

$$Z_{D+s\to D} = 2^{\frac{D_{hori}-D}{D}} \quad \{24\}$$

5.3. Zero-point energy

In this section, we summarize the results about the zero-point energy, ZPE, of *quanta of vacuum*, that are described by the DEQ {19}. Note that these quanta have a tensor structure in general, see e. g. Carmesin (2021a, b), and these quanta include excitation states of vacuum.

At the Planck scale, each quantum has an energy equal to one half of the Planck energy E_P. That energy is equal to the ZPE (see e. g. 2019a, 2021a, 2022c):

$$ZPE(D_{hori}) = \frac{E_P}{2} \quad \{25\}$$

$$\widetilde{ZPE}(D_{hori}) = \frac{1}{2} \quad \{26\}$$

In particular, the *quanta representing the vacuum* and its density ρ_Λ exhibit that ZPE:

$$\widetilde{ZPE}_\Lambda(D_{hori}) = \frac{1}{2} \quad \{27\}$$

5.4. Redshift of quanta of vacuum

In this section, we summarize the results about the redshifts of quanta of the vacuum.

Those quanta of vacuum, that establish space and do not propagate inside space, exhibit a redshift only at a dimensional phase transition, see e. g. Carmesin (2018a, 2019a, 2021a, 2022c). Thereby, the wavelength is increased by the dimensional distance enlargement factor:

$$\lambda_D = \lambda_{D+s} \cdot Z_{D+s\to D} \quad \{28\}$$

Thus, the energy decreases by that factor. Moreover, the energy can change by a polarization factor. For instance, a quantum representing the vacuum in D-dimensional space has one direction of propagation and $D-1$ transverse directions of polarization, so that it fills all D directions of space altogether. So, if the dimension changes from $D+s$ to D, then the number of polarizations is reduced from $D+s-1$ to $D-1$. Thus, the energy is reduced by the following additional polarization factor:

$$q_{D+s\to D,polarization} = \frac{D-1}{D+s-1} \quad \{29\}$$

Thus, the energy of a quantum representing the three-dimensional space as follows:

$$\widetilde{ZPE}_\Lambda(D=3) = \widetilde{ZPE}_\Lambda(D_{hori}) \cdot \frac{2}{300 \cdot Z_{D_{hori}\to 3}} \quad \{30\}$$

The extension of that quantum increases from the Planck length by the factor $Z_{D+s\to D}$. As the density is the energy divided by the volume, the density of three-dimensional vacuum is as follows:

$$\rho_\Lambda(D=3) = \frac{1}{300} \cdot \frac{1}{2^{(D_{hori}-3)\cdot 4/3}} \cdot \frac{E_P}{L_P^3 \cdot c^2 \cdot \frac{4\pi}{3}} \quad \{31\}$$

We calculate that density:

$$\rho_\Lambda(D=3) = 7{,}6 \cdot 10^{-27}\,\frac{kg}{m^3} = \rho_{\Lambda,\text{theo}} \quad \{32\}$$

The observed values differ as a result of the Hubble tension or the local value of the Hubble constant. The value obtained by the Planck satellite is as follows (Planck 2020, Carmesin 2022c):

$$\rho_{\Lambda,\text{obs,Planck}} = 5.93(14) \cdot 10^{-27}\,\frac{kg}{m^3} \quad \{33\}$$

The value obtained by the observation of local galaxies is as follows (Riess 2022, Carmesin 2022c):

$$\rho_{\Lambda,\text{obs,local}} = 8.02(30) \cdot 10^{-27}\,\frac{kg}{m^3} \quad \{34\}$$

Thus, the present theory of quanta of vacuum explains the observed density of vacuum within the errors of measurement. More generally, these quanta form a mixture of wavelengths, a polychromatic vacuum. With it, the time evolution of the density of the vacuum has been derived in precise accordance with observation (Carmesin 2018a, b, 2019a, 2021b, 2022c).

6. Minimal excitation states of vacuum

In this section, we derive minimal excitation states of the vacuum. Thereby, the occupied dimension of a state is minimised so that the polarization is longitudinal, and the energy is minimised.

6.1. Minimal dimensional extension

In this section, we analyse the minimal dimensional extension of an excitation state of vacuum.

The dimensional extension $D_{extension}$ of an excitation state of vacuum is the number of directions, that the direction of propagation plus the polarization directions achieve. For instance, a quantum representing the vacuum in D-dimensional space has the dimensional extension $D_{extension} = D$:

$$D_{extension}[\widetilde{ZPE}_\Lambda(D)] = D \quad \{35\}$$

Thus, a quantum of vacuum achieves the minimal dimensional extension $D_{extension}$, if it has just one longitudinal polarization:

$$D_{extension}[\widetilde{ZPE}_{\text{Long}}(D)] = 1 \quad \{36\}$$

As longitudinal quanta of vacuum exhibit a minimal dimensional extension $D_{extension}$, and thus a minimal dimensional complexity, I proposed that such quanta are relatively likely to form elementary particles (Carmesin 2021b).

6.2. ZPE of longitudinal quanta of vacuum

In this section, we analyse the ZPE of longitudinal quanta of vacuum.

At the Planck scale, at the dimensional horizon, the ZPE is one half of the Planck energy:

$$\widetilde{ZPE}_{\text{Long}}(D_{hori}) = \frac{1}{2} \quad \{37\}$$

Thus, the ZPE at lower dimension is achieved by the redshift as follows:

$$\widetilde{ZPE}_{\text{Long}}(D) = \frac{\widetilde{ZPE}_{\text{Long}}(D_{hori})}{Z_{D_{hori}\to D}} \quad \{38\}$$

Hence, the ZPE at a dimension D is as follows:

$$\widetilde{ZPE}_{\text{Long}}(D) = \frac{1}{2} \cdot 2^{\frac{D_{hori}-D}{D}} \quad \{39\}$$

For instance, a longitudinal quantum of vacuum, that internally has a five-dimensional excitation state, has the following ZPE:

$$\widetilde{ZPE}_{\text{Long}}(D=5) = \frac{1}{2} \cdot 2^{-\frac{301-5}{5}} = 2^{-\frac{301}{5}} \quad \{40\}$$

$\widetilde{ZPE}_{\mathrm{Long}}(D = 5) = 7.551 \cdot 10^{-19}$ {41}

Thus, the ZPE is as follows:

$ZPE_{Long}(D = 5) = 1{,}477 \cdot 10^{-9}$ J {42}

$ZPE_{Long}(D = 5) = 9{,}219$ GeV {43}

6.3. Excitation states of oscillation

In this section, we analyse the excitation states of the oscillation.

Each ZPE corresponds to one half of the circular frequency multiplied by the reduced Planck constant (see e. g. Carmesin 2021a, 2022a, Ballentine 1998):

$ZPE = \frac{1}{2}\hbar\omega$ {44}

The n-th corresponding excitation state is as follows (see e. g. Planck 1911, Carmesin 2021a, 2022a, Ballentine 1998):

$E_n = \left(\frac{1}{2} + n\right)\hbar\omega$ {45}

For instance, the above quantum of vacuum, that internally has a five-dimensional excitation state, has the following corresponding excitation states of oscillation:

$E_{Long,n}(D = 5) = (2n + 1)9{,}219$ GeV {46}

We calculate these values:

$E_{Long,n=1}(D = 5) = 27{,}658$ GeV {47}

$E_{Long,n=2}(D = 5) = 46{,}097$ GeV {48}

$E_{Long,n=3}(D = 5) = 64{,}535$ GeV {49}

6.4. Three-dimensional minimal objects

In this section, we analyse possible three-dimensional minimal objects in three-dimensional space.

Firstly, such a minimal three-dimensional object should consist of minimal objects that are excitation states of vacuum.

Secondly, these minimal objects that are excitation states of vacuum are longitudinal, as only these have dimensional extension $D_{extension}$.

Thirdly, three of these minimal objects form a three-dimensional object, as each object is essentially extended in one dimension only.

Fourthly, these three objects are different, as otherwise they could exhibit a condensation, such as a Bose-Einstein condensation.

Fifthly, a ZPE corresponds to a property of space, similar to ZPE_{Λ}. Correspondingly, $ZPE(D > 3)$ corresponds to higher dimensional space and should not occur in three-dimensional space. I. e., $E_n(D > 3)$ should exist in three-dimensional space for $n \geq 1$ only.

If we apply these five minimal conditions, then we get the following minimal three-dimensional object, that internally has a five-dimensional excitation state:

$object = triple[E_{Long,n=1}(D = 5), E_{Long,n=2}(D = 5), E_{Long,n=3}(D = 5)]$ {50}

Hereby, we mark each excitation state of vacuum by its energy. Thus, the energy of that triple is as follows:

$E = \Sigma_{n=1}^{3} E_{Long,n}(D = 5) = 138{,}3$ GeV {51}

If we apply an additional attractive interaction, we derive the following energy (Carmesin 2021b):

$E_{Triple} = 125{,}4$ GeV {52}

6.5. Mass of the Higgs boson and vev

This result is in accordance with the observed mass of the Higgs boson (Zyla 2020, Carmesin 2021b):

$E_{Higgs,obs} \in [124{,}51\ GeV; 126{,}02\ GeV]$ {53}

In addition, we form a pair of Higgs bosons, hereby we subtract the binding energy. So, we obtain the following energy of the pair as follows (Carmesin 2022b, section 8.7):

$E_{Higgs,pair,theo} = 247{,}6$ GeV = vev {54}

6.6. Masses of W- and Z-bosons

Using that vev, the following masses of the Z- and W-bosons have been derived (Carmesin 2022b, section 8.8):

$M_{Z,theo,here} = 91{,}717$ GeV {55}

$M_{W,theo,here} = 80.409$ GeV {56}

The so-called standard model of elementary particles, SMEP, predicts (see e. g. Altonen et al. 2022):

$M_{W,theo,SMEP} = 80.357(8)$ GeV {57}

Later, a new measurement of the mass of the W-boson has been published (Altonen et al. 2022):

$M_{W,obs} = 80.4335$ GeV$\pm\Delta M$

$with\ \Delta M = 9.4$ MeV {58}

Thus, the new measurement exhibits a difference to the SMEP-value of $8.1 \cdot \Delta M$, while the new measurement exhibits a difference to the present value of only $2.6 \cdot \Delta M$. Moreover, the present derivation does not execute a fit, in contrast to the SMEP.

6.7. Neutrinos

Moreover, the different three-dimensional minimal objects, that are constituted by three longitudinal excitation states, each of which corresponding to three-dimensional space, whereby the energy is minimised, exhibit the sum of the energies of the neutrinos that formed during the time evolution of the universe, for details see Carmesin (2021b).

6.8. Observed phase transition

Furthermore, the three-dimensional minimal objects, that internally have a four-dimensional excitation state, exhibit the energy $E_{object,3D}(D=4) = 4.077 MeV$. The four-dimensional minimal objects, that internally have a four-dimensional excitation state, exhibit the energy $E_{object,4D}(D=4) = 6.522 MeV$. The gravitational wave background, GWB, exhibits a maximum at the following interval of energies, interpreted as a sign of a phase transition, see Ratzinger and Schwaller (2021):

$$E_{GWB} \in [1\ MeV, 10\ MeV]$$

At the last phase transition of the cosmic unfolding from $D = 4$ to $D = 3$, the above objects decay, whereby the energies observed in the GWB are emitted. So, the observed GWB-energies can be explained by the present minimal objects formed by vacuum excitation states, for details, see Carmesin (2021b, section 14.2).

6.9. Elementary charges and couplings

In addition, the minimal three-dimensional objects proposed here have been used to derive the elementary electric charge, the fine structure constant α, the electroweak couplings, the hypercharge, the isospin charge and the masses m_Z and m_W, see Carmesin (2021c, 2022b).

6.10. Quantum physics derived

Moreover, the present theory of vacuum has been used, in order to derive quantum physics. For it, the postulates of quantum physics have been derived in a far distance limit. Quantum physics has been generalized by deriving the physics without that limit, see Carmesin (2022a).

6.11. General relativity derived

Furthermore, the present theory of vacuum has been used, in order to derive general relativity. For it, the Einstein field equation has been derived in a continuum limit, see Carmesin (2022a).

These above results are in precise accordance with observation, whereby these results have been achieved without execution of a fit.

Altogether, these results provide comprehensive evidence for the formation of matter by excitation states of vacuum presented here.

7. Experience with teaching

In this section, I describe experience with teaching in the two different learning groups in more detail.

The present theory has been used in a research club. In the research club, there is a common group consisting of teams, each working on their own project. In this group, there are students from classes five to thirteen. Moreover, there is a subgroup of students interested in quantum gravity and related subjects. That subgroup has an additional meeting each week. At each such additional meeting, we treat one topic in a theoretic and discursive manner. Some of the students of that subgroup start a project in the field of quantum gravity. Usually, the students of that subgroup come from classes nine to thirteen, however, sometimes an exceptional student starts earlier in that subgroup. Many students of the common group, including the subgroup, developed successful projects. So far, ten of the projects of the subgroup achieved an award in a Jugend forscht competition. Some of the students also attend an astronomy club and use the school telescope. Moreover, some students additionally take part in the Herbstakademie, in order to develop their project during one week in the autumn holidays. Furthermore, most students present their results in a public astronomy evening in the assembly hall.

The work at the research club can be concluded as follows: Each student can work at the own individual project and at the own speed. Theoretical foundation and individual aid are both provided. Such conditions enable students to achieve results in topics far beyond the usual curriculum (see e. g. Helmcke, Carmesin, Sprenger and Brüning 2018, Heeren, Sawitzki and Carmesin 2020, Carmesin and Schöneberg 2020, Carmesin and Sawitzki 2021). Moreover, the students can identify limitations of actual knowledge. Furthermore, the students are trained in scientific discussions. Additionally, the students can try to achieve new knowledge on their own. So, they can develop self-esteem efficiently.

In addition, parts of the present theory have been presented at lectures at the university. The lectures have been presented in the framework of general studies. Accordingly, students of various faculties participate. Correspondingly, basic principles are revisited first. On that basis, all results are derived in a comprehensive manner, so that all students can achieve an overview about the chain of arguments ranging from basic principles to advanced results. Additionally, a script is provided. Hereby, several students discussed in a competent manner. Credit points can be achieved on the basis of solved exercises and a well prepared and presented seminar talk.

The work at the lectures can be concluded as follows:

Very different students are interested in the same topic: fundamental physics and cosmology. Progress can be achieved by starting at the physical principles, progressively deriving all results, without a gap in the chain of arguments. However, from the point of view of the students, there are still some gaps in the chain of arguments, or there is some interest to analyse a particular topic in more detail. Such cases are clarified immediately in discussions. Moreover, some items of actual research are intentionally proposed for controversial discussion. The students are supported by a comprehensive script. Such conditions enable students to achieve a substantial progress in learning an advanced topic. Furthermore, many students are or became very competent in scientific discussions. Moreover, the students can identify limitations of actual knowledge and describe ways in which new knowledge has been achieved.

Altogether, the topic is very motivating, fundamental and interesting to students of different age and background.

8. Summary

Leukippos and Democritos proposed two elements (‘stoicheia’), the complete (‘pleres’, matter, consisting of hypothetical indivisible particles named ‘atomos’) and the vacuum (‘kenon’), see Tsaparlis (2001). In modern physics, matter forms from vacuum in the process of a phase transition (Higgs 1964). Thus, the ancient idea of atomism has been developed further by an essential transformation from vacuum to matter. Accordingly, that fundamental transformation provides a substantial didactical perspective, see section 1.

In sections 2 and 3, we analyse, how the idea of that phase transition arose: The **structure** of mass and of fundamental interactions has been investigated experimentally with help of accelerators, for instance (Zyla 2020). The question of the **formation** of mass became pressing, when the principles of least action and of gauge invariance predicted zero mass of the W- and Z-bosons, whereas observations provided huge masses of these bosons. The algebra of the theory enabled a bold hypothesis: mass could form from vacuum via a phase transition – the Higgs (1964) mechanism. Indeed, the Higgs boson has been observed in 2012 (Aad et al. 2012, Chatrchyan et al. 2012, Zyla 2020). However, in the Higgs mechanism, essential questions are not answered: What mechanism of vacuum forms the mass? What spectrum of vacuum is underlying the formation of mass?

In order to use the didactical perspective inherent to the formation of mass from vacuum, an analytic and productive treatment of the subject is essential. For it, we derived answers to the above essential questions:

Firstly, the vacuum consists of quanta that are permanently forming and propagating according to derived differential equations (section 4). These quanta can exhibit phase transitions, whereby the possible states can be derived from the causal horizon, the light horizon (section 5). Minimal objects forming from minimal excitation states of vacuum form the basis of observed masses (sections 6.1 and 6.2).

Secondly, the spectrum of these states is constituted by the phase transitions, by transformations of the symmetry (transverse, longitudinal, tensor), by usual oscillatory excitation states and by the binding of such objects (sections 6.2 – 6.11). Altogether, the present theory provides comprehensive evidence, see sections five and six.

In sections 1.2 and 7, experience with teaching of the present topic is presented. Hereby, the work in a research club and in lectures in general studies is outlined. In both courses, a significant progress in learning and in scientific discussion has been achieved. Thereby, the learners actively treated mathematical and conceptual difficulties: The mathematical difficulties are constituted by the derivations. The learners were enabled to describe and discuss all steps of the derivations, whereby the younger learners needed mathematical introductions additionally, and the advanced learners needed sufficient time, as the derivations have been presented in a complete manner. The description of the formation and propagation of vacuum in terms of differential equations provided no essential conceptual difficulty. Even the well-known hen and egg problem that portions of vacuum generate the space in which they propagate provided no essential difficulty, as it is solved by the formation of vacuum. The phase transition from vacuum to mass according to the Higgs mechanism was not discussed intensively. The dimensional phase transitions had been discussed intensively, whereby the steps of the derivation and the applications have been analysed in detail. This discursive behaviour of the learners shows that the students achieved abilities to analyse and discuss derivations on their own in an advanced manner. Thus, the experience with teaching shows how the didactical perspective of the topic of the phase transition from vacuum to matter can be used by the proposed treatment of the subject.

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