Explanation of Quantum Physics by Gravity and Relativity

- A Possible Course -

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Kurzfassung

Die Quantenphysik ist sehr geeignet zur Beschreibung der Natur und zur Entwicklung neuer Technologie. Allerdings war die Quantenphysik bisher nicht wirklich verstanden. Stattdessen wurden Quantisierungsverfahren und Postulate vorgeschlagen, ohne eine Herleitung aus allgemeineren physikalischen Erkenntnissen zu entwickeln. Nun ist die Quantenphysik als natürliche Folge der Dynamik des Vakuums hergeleitet. Diese Dynamik wiederum ist aus der Gravitation und Relativität hergeleitet. Also ist Quantenphysik eine natürliche Folge von Gravitation und Relativität. In diesem Aufsatz analysiere ich die didaktische Perspektive des Themas. Dazu leite ich die Dynamik des Vakuums sowie die Quantenphysik her. Hierfür schlage ich ein didaktisches Konzept für einen Kurs der Quantenphysik vor. Das Konzept habe ich in zwei Lerngruppen erprobt, in einer Jugend forscht Arbeitsgruppe für die Klassenstufen 8 bis 13 und in einem General Studies Kurs an der Universität Bremen. Ich berichte ich über Erfahrungen mit dem Konzept in den beiden Lerngruppen.

Abstract

Quantum physics is very successful in describing nature and developing technology. However, quantum physics has not yet been really understood. Instead, quantization procedures and postulates had been proposed without derivation from more general physics. Now, quantum physics has been derived as a natural consequence of the dynamics of vacuum. These dynamics, in turn, have been derived from gravity and relativity. Thus, quantum physics is a natural consequence of gravity and relativity. In this paper, I analyse the didactical perspective of the topic. For it, I derive the dynamics of the vacuum, and there from, I derive quantum physics. On that basis, I propose a didactical concept for a course of quantum physics. I tested that concept in two learning groups: a research club for classes 8 to 13 and a general studies course at the university Bremen. I report about the experience with the use of that concept in the two learning groups.

1. Introduction

Students are highly interested in astronomy and astrophysics (Elster 2010, Jenkins 2006, Pössel 2015). Moreover, students are also interested in the characteristics of nature (Elster 2010), this includes the field of quantum physics and general relativity, including gravity. Hereby, Einstein, Podolsky and Rosen (1935) pointed out that the relation between relativity and quantum physics represents an especially interesting question about the characteristics of nature. Indeed, there has been some progress (Bell 1964, Aspect et al. 1981), however, according to Weinberg (2017), that does not yet provide a sufficient answer. Similarly, Feynman (1965, p. 129) wrote: '… I think I can safely say that nobody understands quantum mechanics.' Accordingly, the student's interest in the relation between relativity and quanta provides a basis for a substantial didactical perspective.

1.1. Didactical perspective

The didactical perspective can be analysed in the framework of the didactical triangle, see Franke and Gruschka (1996) or Fig. 1. That triangle represents the relation between the students, the teacher and the subject. Hereby, the relation between the students and the subject is characterized by a high interest of the students, including the motivating aspect of scientific curiosity. According to that scientific question about the relation of relativity and quantum physics, the subject physics can be represented by the two fundamental concepts of physics, see Fig. 2.

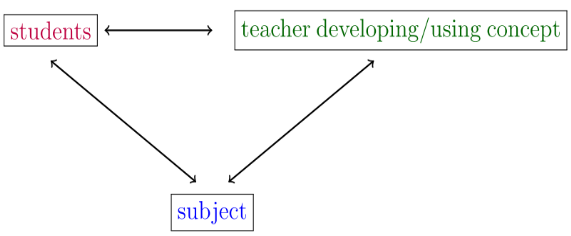

Fig. 1: Didactical triangle.

According to the didactical triangle, the role of the teacher becomes essential. The teacher should take care of the following expectations: Firstly, the students expect answers. Secondly, some advanced students expect methods, so that they can achieve answers on their own. Thirdly, the students expect the teacher to present an actual topic, if possible (see e. g. Busch 2009). The present scientific question represents an actual topic, of course. Fourthly, the teacher is expected to take care of respective curricula. Thus, an actual scientific question can be included in lessons in research clubs, or in university lectures, for instance. Altogether, the present topic has a substantial didactical potential. Hereby, the most interesting didactical perspective is to enable the students to obtain answers on their own. For it, an analysis of the subject and a development of a productive concept are preconditions, see sections (1.2-1.5, 2-6).

Moreover, quantum physics provides substantial opportunities in quantum technologies. So there is an additional didactical potential, whereby the physical and technological aspect are both essential, see Psopiech (2021). In particular, phenomena and basic principles represent essential fields with corresponding skills and competences, see Gerke et al. (2021).

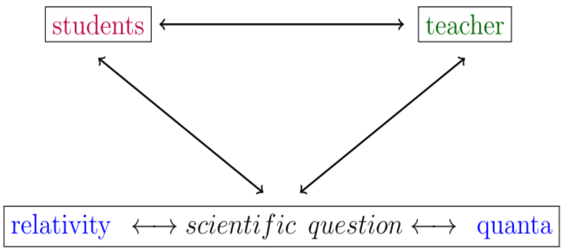

Fig. 2: Didactical triangle with scientific question.

1.2. Didactical perspective of vacuum

The scientific question about the relation of quantum physics and relativity (including gravity) can be treated in an analytic and productive manner. For it, the dynamics of the vacuum are analysed.

These dynamics of vacuum provides several didactical perspectives, see Fig. 3. In fact, I presented the topics illustrated in Fig. 3 to students ranging from class 8 to 13 (see Carmesin, 2021f, Lieber and Carmesin 2021, Carmesin and Carmesin 2020) in a research club and to students of a general studies course at a university. In both groups, the students were able to describe the steps of derivations and to discuss the consequences. Moreover, some advanced students derived some results on their own. Some of these presented their results at Jugend forscht competitions and won prizes. In particular, the students of both groups achieved the following competences:

The students described and discussed the steps of the derivation of the dynamics of vacuum (sections 2-4). As an advance organizer, I summarize the dynamics of vacuum: Vacuum represents a concept of physics, whereas concepts of space are used in mathematics and in physics. Basically, the dynamics of vacuum have been derived from fundamental physical principles including quantum physics, see e. g. Carmesin (2017, 2018a, b, 2019a, 2021a-f), moreover, quantum physics has been derived from gravity and special relativity only (Carmesin 2022a).

In particular, vacuum dynamics provides various results: Basically, the dynamics of vacuum describe the following: three-dimensional vacuum, and more generally higher dimensional vacuum, see e. g. Carmesin (2021a, f). Vacuum dynamics have been analysed at a semi-classical level and at a quantum physical level, see e. g. Carmesin (2021a, f). Basically, vacuum dynamics describe the formation of space, and more generally the formation of matter as well as the formation of elementary charges, couplings and of fundamental interactions, see Carmesin (2021a-f, 2022b, c). Basically, the dynamics of vacuum describe the rate of expansion of space since the Big Bang, including the Hubble constant H_0, and more generally the Hubble tension and the era of inflation (Carmesin 2021a-c). Vacuum dynamics describe the propagation of the gravitational interaction, see Carmesin (2021a). Basically, vacuum dynamics describe the formation and propagation of vacuum at a far distance of a possible black hole with a Schwarzschild radius R_S, $R_S/R>>1$, and more generally the formation and propagation of vacuum in the vicinity of a black hole Carmesin (2022a, Eq. 3.250-3.252).

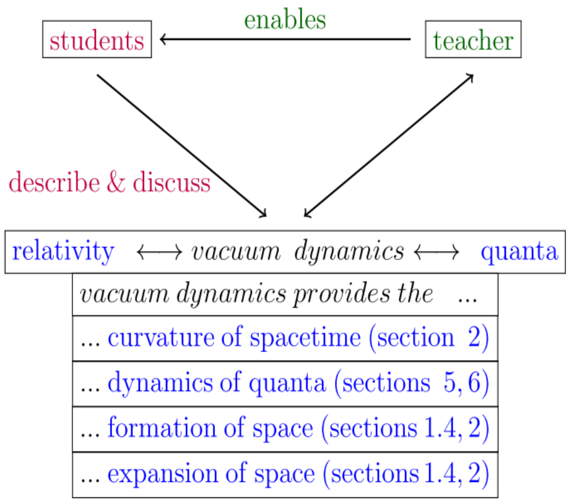

Fig. 3: Didactical triangle illustrates students describing and discussing a derivation of vacuum dynamics as well as items provided by the dynamics of the vacuum.

The students described and discussed how the dynamics of vacuum provides the curvature of spacetime of the Schwarzschild metric (sections 2-4).

Furthermore, the students described and discussed how quanta emerge from the dynamics of vacuum (section 5). Progressively, the students described and discussed how the dynamics of vacuum provides the deterministic and the stochastic dynamics of quanta, including the postulates (sections 5-6). Moreover, the students described and discussed how the dynamics of vacuum provides the formation of space and of its density (section 1.3). Additionally, the students described and discussed how the dynamics of vacuum provides an expansion of space (sections 2-4). Altogether, the didactical potential of the scientific question illustrated in Fig. 2 can be used to generate a course with a substantial didactical perspective, for details see section 9.

1.3. Overview

The scientific question about the relation of quantum physics and relativity (including gravity) provides a substantial didactical potential. In order to reveal this relation, I used gravity and relativity, to derive the dynamics of vacuum (Carmesin 2017, 2018a, b, 2019, 2021a, b), and I used the dynamics of vacuum, to derive quantum physics (Carmesin 2022a, Fig. 4).

Vacuum is characterized by a density, by a spectrum, by a process of formation of vacuum since the Big Bang, by a propagation, by a time evolution, by dimensional phase transitions and by quadrupolar symmetry. Vacuum establishes space, spacetime, curvature of spacetime, the gravitational interaction (similar to the graviton hypothesis), the basic energy of the universe (Carmesin 2020a), objects of vacuum, the formation of elementary particles, the formation of mass, the formation of the elementary charge including the fine structure constant, the formation of electroweak charges and couplings, the formation of quanta. Thus, the dynamics of vacuum provide a natural explanation of quantum physics.

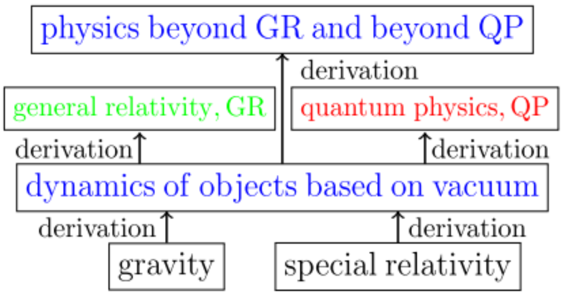

Fig. 4: The dynamics of the vacuum has been derived from gravity and special relativity (Carmesin 2021a). Later it has been discovered for the first time, that vacuum dynamics causes emerging quanta and quantum physics, QP (Carmesin 2022a). Moreover, vacuum dynamics provides general relativity, GR, in a continuum limit (Carmesin 2022b). Furthermore, vacuum dynamics provides physics beyond GR and QP.

1.4. Tests of the dynamics of vacuum

In order to confirm the derived dynamics of vacuum, I elaborated a series of tests: As a first test of that theory of vacuum, I showed that it implies general relativity. (Carmesin 2022b, Figs. 2, 3. For it, I derived the Einstein field equation for an event at a Schwarzschild radius, whereby each event can be represented in a frame so that the event is at a Schwarzschild radius, see Rindler, 1966. Additional quantities, such as charges, can be introduced by using the first law of black hole mechanics, see Bardeen et al. 1973. Moreover, charges have been explained in Carmesin 2021d, Carmesin 2022b.) As a second test, I showed that my dynamics of vacuum provides the density of vacuum (Carmesin 2021a, b). As a third test, I showed the vacuum theory explains the local value of the Hubble constant (Carmesin 2021c, 2022c), that has been observed recently by Riess et al. (2022). As a fourth test, I derived and calculated the six parameters of the standard model of cosmology (Zyla 2020, pp 409-509) on the basis of my dynamics of vacuum (Carmesin 2021b). As a fifth test, I derived and calculated the elementary charge, the fine structure constant and the electromagnetic interactions on the basis of my dynamics of vacuum (Carmesin 2021d). As a sixth test, I derived and calculated the electroweak charges, the weak angle and the electroweak interactions on the basis of my dynamics of vacuum (Carmesin 2022b).

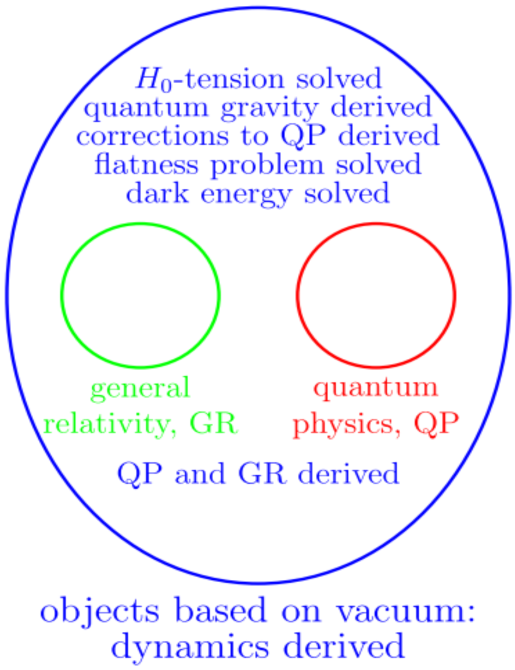

Fig. 5: Relation among physical subject areas: The dynamics of objects based on vacuum is very general (Carmesin 2021a-e). With it, and for particular conditions, quantum physics (Carmesin 2022a) and general relativity (Carmesin 2022b, chapter 9) have been derived.

1.5. Quanta emerging via dynamics of vacuum

In order to understand the nature of vacuum, I analysed properties of my well tested theory of vacuum: Thereby, I discovered that the theory describes a variety of objects (Carmesin 2021b): the present-day vacuum, the vacuum of the early universe, excited states of vacuum, objects propagating at the velocity of light, $v = c$, as well as objects propagating slower, $v < c$. Each object that is described by my theory of vacuum is called an object of vacuum. Thus, the objects of vacuum include the above variety of objects. Moreover, the objects of vacuum include elementary particles, electric charges and electroweak charges or couplings (Carmesin 2021d, Carmesin 2022b).

Moreover, I discovered that all observable objects of vacuum are quanta (Carmesin 2022a). Furthermore, I derived the following: If a distant observer measures properties of these quanta, then these properties can be described by the postulates of quantum physics. In particular, I derived the postulates of quantum physics from the dynamics of vacuum. Thus, quantum physics is a natural property of the observable objects of vacuum.

Hence, general relativity and quantum physics can be derived from my theory of the vacuum. This is an essential relation between general relativity and quantum physics. Moreover, the postulates of quantum physics have become consequences of the more fundamental theory of vacuum. In this paper, I propose a course that uses the above fundamental theory or dynamics of vacuum.

1.6. Postulates

Quantum physics is usually described by a set of postulates (Kumar 2018 or Ballentine 1998). Accordingly, the concept of a conventional course of quantum physics is to motivate the postulates by experiments.

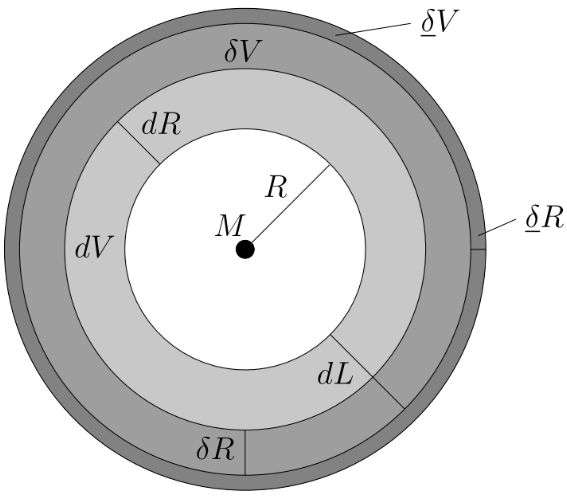

Fig. 6: Formation of vacuum caused by a mass M.

1.7. Proposed course

Here, a more fundamental course is proposed. Essential steps of the course are as follows:

- The following might be known from school: relativity (Burisch et al., 2022, p. 472-490), gravity (Carmesin et al., 2021, p. 102-120)
- Most easily, the dynamics of vacuum are derived from the Schwarzschild metric.
- The Schrödinger (1926) equation is directly derived from the dynamics of vacuum.
- The postulates of quantum physics are derived from the dynamics of vacuum.

2. Formation of vacuum

In this section, I show how a mass M with a Schwarzschild radius $R_S = 2GM/c^2$ forms vacuum at a coordinate distance R from M. Hereby, $G = 6.67430(15) \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$ is the gravitational constant, and $c = 299\ 792\ 458 \frac{m}{s}$ is the velocity of light (Zyla 2020). In particular, I derive the corresponding differential equation, DEQ. I derived this result directly from gravity and relativity (Carmesin 2022a, section 3.9). Here I present a shorter derivation based on the Schwarzschild metric (Fig. 6).

A mass M with a Schwarzschild radius $R_S = 2GM/c^2$ causes curvature of spacetime (Schwarzschild 1916 or e. g. Carmesin 1996). The radial component of that curvature represents the elongation of a radial coordinate difference dR to a physical length dL as follows:

$$dL = dR \cdot \left(1 - \frac{R_S}{R}\right)^{-\frac{1}{2}} \qquad \{1\}$$

Hereby, R is the radial coordinate with the mass at $R = 0$ and dR, δR as well as $\underline{\delta} R$ are radial coordinate differences at R (Fig. 6). I emphasize that R, dR, δR and $\underline{\delta} R$ can be measured by a local observer, see e. g. Carmesin (2022a, p. 19). Also dL can be measured, for instance, the time of flight can be observed. Thus, these quantities are as real physical quantities can be.

Differences marked by d, δ or $\underline{\delta}$ are regarded as infinitesimal in the sense of the Leibniz (1684) calculus, so that corresponding derivatives are exact. Next, we analyse the elongated length dL in the limit of limit R_S/R to zero, we name that limit the far distance limit. For it, we apply the first order Taylor approximation:

$$dL \doteq dR \cdot \left(1 + \frac{R_S}{2R}\right) \qquad \{2\}$$

Thus, the observable increase of length is as follows:

$$\delta R = dR \cdot \frac{R_S}{2R} \qquad \{3\}$$

Hence, the observable increase of length δR corresponds to an observable increase of volume δV as follows:

$$\delta V = 4\pi R^2 \delta R = 2\pi R_S R \cdot dR \qquad \{4\}$$

I emphasize that this increase of volume is filled with additional vacuum, whereby the vacuum has a vacuum density ρ_Λ or ρ_v (Perlmutter et al. 1998, Riess et al. 2000). Hereby, that vacuum corresponds to 68.47 % of all energy or mass in the universe (Planck collaboration 2020). Thus, the additional vacuum is as real as a physical quantity can be.

Moreover, that additional vacuum propagates at the velocity of light. Otherwise, an object's velocity relative to the additional vacuum (with volume δV or space with volume δV) could be measured. However, a relative velocity relative to space cannot be measured, according to relativity. So, the question arises, whether this additional vacuum (with volume δV) is formed at M, at R or anywhere else. This question is analysed next.

Corresponding to the symmetry, the additional vacuum flows radially away from the mass M. During a time-interval $\underline{\delta}t$, the vacuum δV propagates a radial difference $\underline{\delta}R = c \cdot \underline{\delta}t$ (Fig. 6). We analyse the difference $\underline{\delta}V$ of the vacuum δV, that occurs during that propagation. For it, we multiply the partial derivative ∂_R of δV with the difference $\underline{\delta}R$:

$$\underline{\delta}V = \partial_R \, \delta V \cdot \underline{\delta}R \qquad \{5\}$$

We apply equation {4}:

$$\underline{\delta}V = 2\pi R_S R \cdot dR \ \cdot \underline{\delta}R \qquad \{6\}$$

We use $dV = 4\pi R^2 dR$:

$$\underline{\delta}V = \frac{R_S}{2R} \cdot dV \ \cdot \underline{\delta}R \qquad \{7\}$$

We apply the Schwarzschild radius $R_S = \frac{2GM}{c^2}$:

$$\underline{\delta}V = \frac{GM}{R^2} \cdot dV \ \cdot \frac{\underline{\delta}R}{c^2} = \frac{GM}{R^2} \cdot dV \ \cdot \frac{\underline{\delta}t}{c} \qquad \{8\}$$

We identify the first fraction with the gravitational field at R:

$$G^*(R) = \frac{GM}{R^2} \qquad \{9\}$$

Thus, the new volume $\underline{\delta}V$ that formed from the present volume dV during a time $\underline{\delta}t$ is as follows:

$$\underline{\delta}V = \frac{G^*}{c} \cdot dV \ \cdot \underline{\delta}t \qquad \{10\}$$

We name the ratio of the new vacuum $\underline{\delta}V$ formed from the present vacuum dV by the relative formation ε:

$$\varepsilon = \frac{\underline{\delta}V}{dV} \qquad \{11\}$$

During that process, the present volume dV is constant. Thus, within the Leibniz calculus, the time derivative of the relative formation ε is as follows:

$$\dot{\varepsilon} = \frac{\underline{\delta}V}{dV \cdot \underline{\delta}t} \qquad \{12\}$$

This time derivative represents the (relative) rate of formation of vacuum. We solve equation {10} for the rate of formation of vacuum:

$$\dot{\varepsilon} = \frac{G^*}{c} \qquad \{13\}$$

This DEQ describes the formation of vacuum caused by a mass M. Many properties of that formed vacuum have been analysed in detail in Carmesin (2021a).

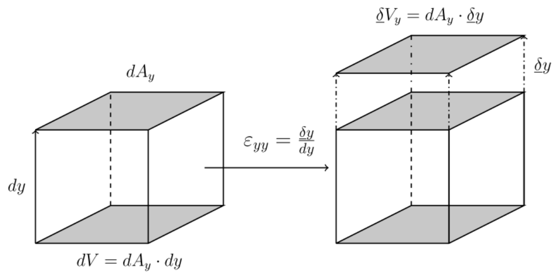

Fig. 7: Formation of vacuum caused by a mass M and in a direction y.

3. Tensors

In this section, we show how Eq. {13} is generalized to tensors.

A mass M causes the formation of vacuum in a radial direction, for instance, the radial direction can be marked by y (Fig. 7). Accordingly, the relative formation is the ratio of $\underline{\delta}y$ and dy, correspondingly, that relative ratio can be described by a tensor element ε_{yy} as follows:

$$\varepsilon_{yy} = \frac{\delta y}{dy} \qquad \{14\}$$

That tensor describes the formation of vacuum in one direction, it is called unidirectional formation of vacuum (Carmesin 2021a). That tensor also describes the fact that the formed vacuum flows away from the mass M in a radial manner or in direction y in Fig. 7.

If the same unidirectional formation of vacuum takes place in each Cartesian direction, then isotropic formation of vacuum takes place (Carmesin 2021a):

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} \quad and \quad \varepsilon = \Sigma_{j=1}^{j=3} \varepsilon_{jj} \qquad \{15\}$$

A nondiagonal element of the tensor can be interpreted by a deformation (Fig. 8):

$$\varepsilon_{i,j} = \frac{\delta r_i}{dr_j} \qquad \{16\}$$

Such deformations can occur in spacetime, see e. g. Carmesin (2021a, section 5.4). Additionally, nondiagonal tensors can be obtained from diagonal tensors by coordinate transformations.

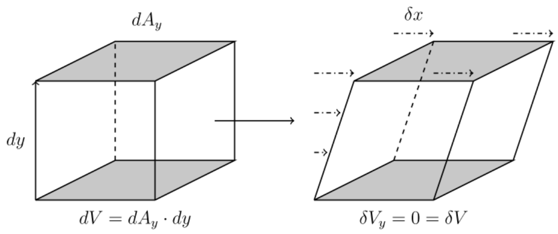

Fig. 8: Deformation $\delta x = \varepsilon_{x,y} \cdot dy$ of vacuum.

For the case of a diagonal element of the rate in Eq. {13}, the gravitational field is parallel to the direction of the rate, and so the formation of vacuum is described as follows:

$$\dot{\varepsilon}_{jj} = \frac{G_j^*}{c} = \dot{\varepsilon}_j \qquad \{17\}$$

Hereby, we abbreviated $\dot{\varepsilon}_{jj} = \dot{\varepsilon}_j$. For the case of a nondiagonal element of the rate in Eq. {13}, we introduce the gravitational field tensor. For it, we express the field G_j^* as a force per probing mass m:

$$G_j^* = \frac{F_j}{m} \qquad \{18\}$$

Additionally, we describe the force by the product of stress and area A (Fig. 8), see Sommerfeld (1978):

$$G_j^* = \frac{\sigma_{jj} \cdot A}{m} \qquad \{19\}$$

Thus, the gravitational field tensor is obtained by realizing that the tensor property of the stress tensor is transferred to the gravitational field:

$$G_{jj}^* = \frac{\sigma_{jj} \cdot A}{m} \quad \text{and} \quad G_{ij}^* = \frac{\sigma_{ij} \cdot A}{m} \qquad \{20\}$$

Thus, the formation of vacuum is described as follows:

$$\dot{\varepsilon}_{ij} = \frac{G_{ij}^*}{c} \qquad \{21\}$$

4. Propagation of vacuum

The dynamics of the formation of vacuum (Eqs. {13} or {17}) causes the dynamics of the propagation of vacuum (Carmesin 2021a, chapter 5). In this section, we derive the propagation for the case of plane waves. For it, we apply the gravitational potential ϕ to Eq. {17}:

$$\dot{\varepsilon}_j = \partial_t \varepsilon_j = -\frac{1}{c} \cdot \partial_{r_j} \phi \qquad \{22\}$$

The following plane waves are solutions of the above DEQ:

$$\varepsilon(t, R_j) = \hat{\varepsilon}_{j,\omega} \cdot \sin(\omega \cdot t - k_j \cdot R_j) \qquad \{23\}$$

$$\phi(t, R_j) = \hat{\phi}_{j,\omega} \cdot \sin(\omega \cdot t - k_j \cdot R_j) \qquad \{24\}$$

We insert into the DEQ:

$$\hat{\varepsilon}_{j,\omega} \omega \cos(\omega t - k_j R_j) = \hat{\phi}_{j,\omega} \frac{k_j}{c} \cos(\omega t - k_j R_j)$$

We subtract the right-hand side of the above Eq., and we factorize:

$$\cos(\omega t - k_j R_j) \cdot \left(\hat{\varepsilon}_{j,\omega} \omega - \hat{\phi}_{j,\omega} \frac{k_j}{c} \right) = 0$$

As the cosine can take nonzero values, the large bracket is zero:

$$\hat{\varepsilon}_{j,\omega} \omega - \hat{\phi}_{j,\omega} \frac{k_j}{c} = 0 \qquad \{25\}$$

These dynamics describes waves that propagate at arbitrary velocity:

$$v = \frac{\omega}{k_j} \qquad \{26\}$$

With it, we solve Eq. {25}:

$$\hat{\phi}_{j,\omega} = \hat{\varepsilon}_{j,\omega} \cdot c \cdot v \qquad \{27\}$$

With it, we express the wave of the potential (Eq. {24}) by the rate:

$$\phi(t, R_j) = \hat{\varepsilon}_{j,\omega} \cdot c \cdot v \cdot \sin(\omega \cdot t - k_j \cdot R_j) \quad \text{or}$$

$$\phi(t, R_j) = \varepsilon(t, R_j) \cdot c \cdot v \qquad \{28\}$$

Thus, the propagation can be described by the rate $\varepsilon(t, R_j)$ only. Thence, objects of vacuum propagate in the form of waves. These waves are characterized by the rate $\varepsilon(t, R_j)$ and by gravity, in the form of the gravitational potential $\phi(t, R_j)$. Accordingly, these waves are called rate gravity waves, RGW (Carmesin 2021a, chapter 5).

So far, we showed that all harmonic plane waves are solutions of the DEQ {22}. As that DEQ is linear, all linear combinations of all harmonic plane waves are solutions. So, the solutions of the DEQ establish a Hilbert space H, whereby we apply the usual scalar product (Carmesin 2022a, section 3.12.1):

$$\langle \psi_1 | \psi_2 \rangle = \int \psi_1(x) \cdot \psi_2^{cc}(x) dx \qquad \{29\}$$

Herby, ψ_2^{cc} marks the complex conjugate of ψ_2^{cc}.

4.1. Objects with a rest mass

All solutions of DEQ {22} represent objects of vacuum. Thus, there are objects of vacuum that propagate at a velocity $v = c$, and there are objects that propagate at $v < c$. Objects propagating at $v < c$ have a nonzero rest mass $m_0 \neq 0$, whereas objects propagating at $v = c$ have zero rest mass $m_0 = 0$, according to relativity (Einstein 1905, Carmesin 2021b, 2022a).

4.2. Observable objects of vacuum

Masses typically are excitations of vacuum. This is the essence of the Higgs (1964) mechanism. The Higgs mechanism explains the formation of mass by a postulated phase transition, and the corresponding Higgs particle has been discovered (Aad et al. 2012, Chatrchyan et al. 2012). The dynamics of vacuum naturally exhibit phase transitions that explain the formation of a variety of masses, whereby the values of the masses are provided in addition, and whereby the mechanisms of formation of electric or weak charges or couplings are provided also, including the precise values of these charges and couplings (Carmesin 2021b, d, Carmesin 2022b). Thus, elementary particles with nonzero rest mass are objects of vacuum that can be observed.

Furthermore, there are objects of vacuum with zero rest mass, such as objects presenting vacuum in three-dimensional space. For such objects, I derived the energy spectrum by two very different methods: I used the time evolution starting at the Planck scale (Carmesin 2018a, b, 2019), or I used the time evolution starting at present-day vacuum (Carmesin 2021a, chapter 4, Carmesin 2022c).

5. Emerging quanta

In this section, we show how objects based on vacuum exhibit quanta as a result of the dynamics of the vacuum.

5.1. Quanta of objects with v = c

In this section, we show how relativistic objects based on vacuum are inherently quantized. Hereby, we do not yet include an interaction.

A relativistic object propagating in a direction x can be characterized by its energy E and by its nonzero momentum p_x. Hereby, the following relativistic relation holds (Einstein 1905):

$$\frac{E}{p_x} = c \qquad \{30\}$$

As the object is based on vacuum and as it propagates at the velocity of light, $v = c$, it can be described by a wave with a circular frequency ω and a wave vector component k_x. Hereby, the following relation holds:

$$\frac{\omega}{k_x} = c \qquad \{31\}$$

So, the above fractions are equal:

$$\frac{\omega}{k_x} = \frac{E}{p_x} = c \qquad \{32\}$$

As ω is nonzero, we can derive the following relation:

$$\frac{E}{\omega} = \frac{p_x}{k_x} \neq 0 \qquad \{33\}$$

The two fractions represent the same positive function or constant $K(\omega)$. That term $K(\omega)$ cannot depend on time, as there is no interaction:

$$\frac{E}{\omega} = \frac{p_x}{k_x} = K(\omega) \qquad \{34\}$$

In order to show the universality of that term, we analyse the gravitational redshift of an object with $v = c$, energy E, at a coordinate distance R from a mass M, and we show that $K(\omega)$ is the same for all ω. We denote $E_\infty = lim_{R\to\infty} E(R)$, $\omega_\infty = lim_{R\to\infty}\omega(R)$. Additionally, the energy observed by a local observer at R is named $E_{obs,loc}(R)$. Thus, at R the position factor $\varepsilon(R) = \sqrt{1 - R_S/R}$ describes the decrease of energy according to position. According to the law of energy conservation, E_∞ is a constant, and hence we derive:

$$E_\infty = E_{obs,loc}(\mathrm{R}) \cdot \varepsilon(R) \qquad \{35\}$$

We solve for $E_{obs,loc}(R)$:

$$E_{obs,loc}(R) = \frac{E_\infty}{\varepsilon(R)} \qquad \{36\}$$

We apply the quantization Eq. {34}:

$$E_{obs,loc}(R) = K(\omega(R)) \cdot \omega(R) \qquad \{37\}$$

We use the Schwarzschild metric (Fig. 6):

$$\omega(\mathrm{R}) = \frac{\omega_\infty}{\varepsilon(\mathrm{R})} \qquad \{38\}$$

We analyse two observers at R and R':

$$\frac{E_{obs,loc}(R)}{E_{obs,loc}(R')} = \frac{K(\omega(R))\omega(R)}{K(\omega(R'))\omega(R')} \qquad \{39\}$$

We apply Eqs. {36} and {38}:

$$\frac{E_\infty \varepsilon(R)}{E_\infty \varepsilon(R\prime)} = \frac{K(\omega(R))\omega_\infty \varepsilon(R)}{K(\omega(R'))\omega_\infty \varepsilon(R\prime)} \qquad \{40\}$$

We simplify that equation:

$$1 = \frac{K(\omega(R))}{K(\omega(R'))} \qquad \{41\}$$

Thus, the quantization constant does not depend on ω, and according to Eq. {31}, that constant does not depend on the wave number. Thence, the constant K is a universal constant. That constant can be measured, and it takes the value of the reduced Planck constant $\hbar$:

$$K = \hbar = h/2\pi \qquad \{42\}$$

We insert equation {42} into equation {34}, in order to derive the usual relations of quantization of energy and momentum:

$$E = \hbar \cdot \omega \qquad \{43\}$$

$$p_x = \hbar \cdot k_x \qquad \{44\}$$

5.2. Derivation of the Schrödinger equation

In this section, we use the dynamics of the vacuum, in order to derive the Schrödinger equation.

For it, we apply the DEQ {22}, whereby we substitute the potential by the rate according to Eq. {28}, and we consider the case $v = c$:

$$\partial_t \varepsilon_j = -c \cdot \partial_{r_j} \varepsilon_j \qquad \{45\}$$

In order to obtain the correct wave dynamics and stochastic dynamics, we apply the time derivative to the above Eq. (Carmesin 2022a, section 3.8.3):

$$\partial_t \dot{\varepsilon}_j = -c \cdot \partial_{r_j} \dot{\varepsilon}_j \qquad \{46\}$$

In order to get a simple notation, we name R_j by x:

$$\partial_t \dot{\varepsilon} = -\partial_x \dot{\varepsilon} \cdot c \qquad \{47\}$$

In order to derive the traditional form of the Schrödinger equation, we multiply by $i\hbar$, we multiply the rate $\dot{\varepsilon}$ by a normalization factor t_n, and we use the product as the wave function ψ:

$$i \cdot \hbar \cdot \partial_t \dot{\varepsilon} = -i \cdot \hbar \cdot \partial_x \dot{\varepsilon} \cdot c \qquad \{48\}$$

$$\dot{\varepsilon} \cdot t_n = \psi \qquad \{49\}$$

$$i \cdot \hbar \cdot \partial_t \psi = -i \cdot \hbar \cdot \partial_x \psi \cdot c \qquad \{50\}$$

Next, we compare the resulting DEQ with the relativistic relation between energy E of the object and the momentum p of the object:

$$E = p \cdot c \qquad \{51\}$$

An object of vacuum is described by the DEQ {50} and by equation {51}, the factors in front of the wave function can be identified by the operators corresponding to the energy E and the momentum p as follows:

$$\hat{E} = i\hbar\partial_t \qquad \{52\}$$

$$\hat{p}_x = -i\hbar\partial_x \qquad \{53\}$$

So, the DEQ {50} describing objects of vacuum with $v = c$ takes the following form:

$$\hat{E}\,\psi = c \cdot \hat{p}\,\psi \qquad \{54\}$$

In the above equation, the product $c \cdot \hat{p}$ represents the energy function $E(p)$ as a function of the momentum, whereby the momentum is represented by its operator:

$$\hat{E}\,\psi = E(\hat{p})\,\psi \qquad \{55\}$$

In a considered physical system, the energy function can also depend on a potential V or on a rest mass m_0 or on a velocity v, or on an additional physical quantity q, for instance. Accordingly, these terms are included in the energy function in equation {55}:

$$\hat{E}\,\psi = E(\hat{p}, V, m_0, v, q)\,\psi \qquad \{56\}$$

This equation represents a usual form of a Schrödinger equation, SEQ: The energy operator multiplied by the wave function is equal to the energy function multiplied by the wave function.

5.3. Quanta of objects with v < c

In this section, we use the fact that objects based on vacuum are described by the differential equation, DEQ, {22}. Thereby, this DEQ provides solutions that propagate with a velocity v, whereby $v = c$ as well as $v < c$ are possible. In this section, we show that objects based on vacuum and propagating at a velocity $v < c$ are inherently quantized. Hereby, we do not yet include an interaction.

For an object based on vacuum, the DEQ {22} holds. As a consequence, derived in section 5.2, the SEQ {56} holds for such an object:

$$\hat{E}\,\psi = E(\hat{p}, V, m_0, v, q)\,\psi \qquad \{57\}$$

Next, we analyse the form of the energy function for the case of an object of vacuum propagating at a velocity $v < c$ and without interaction. An object propagating in a direction x can be characterized by its energy E and by its nonzero momentum p_x. As the object propagates at a velocity $v < c$, it has a rest mass or own mass m_0 (Einstein 1905). Correspondingly, the following relation holds:

$$E^2 = p_x^2 \cdot c^2 + m_0^2 \cdot c^4 \qquad \{58\}$$

With it, the SEQ {57} takes the following form:

$$\hat{E}\,\psi = \sqrt{\hat{p}_x^2 \cdot c^2 + m_0^2 \cdot c^4}\,\psi \qquad \{59\}$$

This is the SEQ of an object of vacuum propagating at a velocity $v < c$ and without interaction.

In particular, if the object of vacuum propagating at a velocity $v < c$ and without interaction so that $p_x^2/(m_0^2 \cdot c^2)$ is very small compared to one, then the linear approximation of the SEQ {59} is appropriate:

$$\hat{E}\,\psi \doteq m_0 c^2 \cdot \left(1 + \frac{1}{2} \cdot \frac{\hat{p}_x^2}{m_0^2 c^2}\right)\psi \qquad \{60\}$$

We simplify:

$$\hat{E}\,\psi \doteq m_0 c^2 \cdot \psi + \frac{\hat{p}_x^2}{2m_0}\,\psi \qquad \{61\}$$

This is the SEQ of an object of vacuum propagating at a velocity $v < c$, without interaction and with a relatively large rest mass.

As the rest mass is relatively large, the factorization outlined in section 4.3 may be applied. Thus, the above SEQ is separated as follows:

$$\hat{E}_0\,\psi_0 \doteq m_0 c^2 \cdot \psi_0 \qquad \{62\}$$

$$\hat{E}_1\,\psi_1 \doteq \frac{\hat{p}_x^2}{2m_0}\,\psi_1 \qquad \{63\}$$

The SEQ {63} is the usual SEQ for a nonrelativistic object without interaction.

6. Background information

In this section, I describe the derivation of the postulates of quantum physics. Hereby, I introduce mixed states. Additionally, I provide further background information.

6.1. Postulate on quantum states

A first postulate is as follows: A quantum state is described by a vector in a Hilbert space (Kumar 2018, p. 168).

That Hilbert space is established by the solutions of the DEQ {22}. For a detailed derivation of that postulate, see Carmesin (2022a, section 3.12.1).

6.2. Postulate on observables

A second postulate is as follows: An observable A is represented by a hermitian or self-adjoint operator $\hat{A}$ acting in the Hilbert space H (Kumar 2018, p. 169).

We derive that postulate as follows: An object that acts upon a state in Hilbert space is an operator. A measurement apparatus providing an observable A acts upon a state in reality. Thus, a measurement apparatus of an observable A corresponds to a linear operator $\hat{A}$ acting in the Hilbert space H of the object or system under consideration.

Moreover, the measurement apparatus can provide a value of the observable A, without changing the state. For instance, a polarizer measures polarization, and a state with the corresponding polarization is transmitted. Thus, there should be states that are not changed by the operator $\hat{A}$, these are the eigenstates, each with a corresponding eigenvalue, e. g. a_j.

Furthermore, the results of such a single measurement are represented by a real number. Thus, each operator $\hat{A}$ corresponding to an observable A has real eigenvalues only. Thence, that operator is self-adjoint, as only self-adjoint operators only have real eigenvalues (Teschl 2014).

Altogether, the second postulate is derived. For details see Carmesin (2022a, section 3.12.2).

6.3. **Postulate on outcomes of measurements**

A third postulate is as follows: A possible outcome of a measurement of an observable A is an eigenvalue of $\hat{A}$ (Kumar 2018, p. 169).

As derived in the above section, an observable A and its measurement apparatus are represented by a self-adjoint operator $\hat{A}$. It is characterized by a spectrum of eigenvalues a_j with corresponding eigenstates ψ_j (Teschl 2014, theorem 3.6). Correspondingly, these eigenstates correspond to possible outcomes of measurements, for details see Carmesin (2022a, section 3.12.3).

6.4. **Postulate on probabilistic outcomes**

A fourth postulate about probabilistic outcomes of measurements is as follows, whereby we present the case of discrete and non-degenerate eigenvalues (Kumar 2018, pp 169-170):

'If a measurement of an observable A is made in a normalized state $|\psi(t)\rangle$ of the quantum mechanical system, then the following holds:

The probability of obtaining one of the non-degenerate discrete eigenvalues a_j of the corresponding operator $\hat{A}$ is given by:'

$$P(a_j) = |\langle\phi_j|\psi\rangle|^2 \qquad \{64\}$$

That result has been derived for the case of the objects of vacuum in Carmesin (2022a, section 3.12.4). Thereby, the following more basic result has been derived and applied: The square of the field $G^*(x)$ is proportional to the energy density:

$$u_f(x) \propto |G^*|^2(x) \qquad \{65\}$$

That square is proportional to the square of the rate $\dot{\varepsilon}^2$, which is in turn proportional to the absolute square of the wave function $|\psi(x)|^2$. Thus, we derive:

$$u_f(x) \propto |\psi(x)|^2 \qquad \{66\}$$

The probability $p(x)$ to measure an object of vacuum at a location x is proportional to the energy density, as the energy is quantized. Thence we obtain the proportionality of the probability and the absolute square of the wave function:

$$p(x) \propto u_f(x) \propto |\psi(x)|^2 \qquad \{67\}$$

6.5. **Postulate on the dynamics**

The dynamics of quanta can be represented by the Schrödinger equation (Kumar 2018, p. 170). We derived the result in section 5, for details see Carmesin (2022a, section 3.12.5).

6.6. **Postulate on mixed states**

The above postulates describe the physics of states that correspond to a one-dimensional subspace of Hilbert space. In this section, we analyse mixed states that correspond to a higher dimensional subspace of Hilbert space. For instance, Ballentine (1998, p. 46) or Grawert (1977) present such postulates.

Firstly, we show that objects of vacuum typically exhibit such mixed states. A pure state represents a coherent wave function, as a pure state is represented by a one-dimensional subspace of Hilbert space. We consider two equal masses m_1 and m_2. These masses do permanently generate vacuum. If each mass would generate that vacuum in the form of a coherent wave function, then these wave functions would form a standing wave in the space between the two masses. Hence an observer could determine his position and velocity relative to a node of the standing wave. Thence the observer could determine his velocity relative to space, however, that is not possible according to relativity. Thus, the masses generate the vacuum in terms of several independent waves in an incoherent manner. So, the generated vacuum is represented by a mixed state.

Secondly, we derive the probabilistic properties of such mixed states, and we represent these properties in the usual manner (Ballentine 1998, Grawert 1977):

In order to analyse a mixed state, we use an operator $\hat{A}$ of an observable A. Accordingly, a mixed state consists of several eigenstates $\langle\phi_j|$, each with an eigenvalue a_j and with a probability p_j. Thus, the expectation value of a measurement is obtained by the weighted sum of the eigenvalues:

$$\langle\hat{A}\rangle = \Sigma_j a_j p_j \qquad \text{with} \qquad 1 = \Sigma_j p_j \qquad \{68\}$$

We identify the sum with the following trace:

$$\langle\hat{A}\rangle = Tr(\Sigma_j |\phi_j\rangle\, p_j\, a_j \langle\phi_j|) \qquad \{69\}$$

We add an additional sum that does not change the value of the term:

$$\langle\hat{A}\rangle = Tr\left(\Sigma_i\, \Sigma_j |\phi_i\rangle p_i\, \delta_{ij}\, a_j \langle\phi_j|\right) \qquad \{70\}$$

We represent the Kronecker delta by a scalar product of the orthonormal eigenfunctions:

$$\langle\hat{A}\rangle = Tr(\Sigma_i\, |\phi_i\rangle p_i\, \langle\phi_i|\, \Sigma_j\, |\phi_j\rangle\, a_j \langle\phi_j|) \qquad \{71\}$$

We identify the sum by a linear combination of projection operators $\hat{P}_i$:

$\Sigma_i |\phi_i\rangle\, p_i \langle\phi_i| = \Sigma_i\, p_i \hat{P}_i = \hat{\rho}$ {72}

The above linear combination of projection operators is called density operator. We identify the density operator and the operator $\hat{A} = \Sigma_j |\phi_j\rangle\, a_j \langle\phi_j|$ in Eq. {71}:

$\langle\hat{A}\rangle = Tr(\hat{\rho}\, \hat{A})$ {73}

Thus, the expectation value $\langle\hat{A}\rangle$ of a measurement of an observable A of a mixed state described by a density operator $\hat{\rho}$ is equal to the trace of the product of the density operator $\hat{\rho}$ and the operator $\hat{A}$. This sentence represents the postulate about mixed states, see Ballentine (1998, p. 46).

Altogether, we derived all six postulates of quantum physics.

6.7. On generalizations

So far, we analysed plane waves and a corresponding one-dimensional version of quantum physics. A three-dimensional generalization is straight forward. Thereby, the spin enters as a consequence. Additionally, the spin statistics theorem enters as a further consequence. Furthermore, the second quantization can be applied (Carmesin 2021a, chapter 6). With it, a quantum field theory can be derived.

Moreover, additional interactions can be derived by using the principle of gauge invariance (Carmesin 2022b, chapter 8). For it, the elementary charges are fundamental. These can be derived by analysing and using phase transitions of vacuum (Carmesin 2021d).

7. Nature of quantum gravity

Many approaches to quantum gravity have been published. E.g., Chandrasekhar (1931) used laws of gravity and laws of quantum physics, in order to derive the mass at which a white dwarf becomes unstable. Kiefer (2003) discussed various methods of combining gravity and quantum physics. Giulini (2003) discussed various methods of quantization of a relativistic or non-relativistic classical description. In all these approaches, a classical description has been combined with methods of quantum physics.

Now, we discover how quantum gravity emerges in a completely natural manner from the well tested dynamics of the vacuum: For it, we generalized general relativity, in order to obtain the dynamics of vacuum. Then we discovered that the dynamics of vacuum inherently makes possible a far distance limit, which naturally provides quantum physics of relativistic ($v = c$) and massive ($v < c$) observable objects.

Thus, if we do not perform the far distance limit, or if we use correction terms corresponding to the far distance limit, then we obtain a generalization of quantum physics. That generalization provides a natural theory of quantum gravity (Carmesin 2022a).

8. Proposed course

In this section, we propose a course of quantum physics that uses the fundamental dynamics of the vacuum. For all lessons, material for exercises and examples can be found in Carmesin (2022a).

8.1. Recapitulation

In a first lesson, the law of gravity, special relativity and the Schwarzschild metric should be recapitulated.

In order to avoid unnecessary complications, I recommend a presentation provided by school books:

Firstly, Newton's law of gravity should be treated (Carmesin et al., 2021, p. 102-120). Hereby, I recommend to mention that the $1/r^2$-law has an empirical basis, whereas Newton's speculations about absolute time and space are hypothetic.

Secondly, special relativity should be recapitulated (Burisch et al., 2022, p. 472-483).

Thirdly, gravity and special relativity can be applied, in order to derive the Schwarzschild metric (Burisch et al., 2022, p. 484-490).

8.2. Formation of vacuum

In a second lesson, the Schwarzschild metric should be applied, in order to derive the DEQ {13} describing the formation of vacuum, see section 2.

8.3. Tensors

In a next lesson, tensors describing the formation vacuum and possible deformations can be treated. Thereby, the following lesson on propagation is prepared. Moreover, there is time for some exercises.

8.4. Propagation of vacuum

Based on lessons two and three, the propagation of vacuum should be derived, see section 4. Hereby the DEQ {13} is applied, in order to derive plane waves. Hereby, the students can develop main results on their own.

8.5. Objects of vacuum

Based on the propagation of vacuum, relativistic and nonrelativistic objects of vacuum can be derived. Hereby, there should be time for exercises.

8.6. Hilbert space

Based on the propagation of plane waves and on the linear DEQ, the space of solutions, the Hilbert space, should be introduced. Depending on the students and on the available time, the correspondence between observables and hermitian operators should be introduced and analysed.

8.7. Discovery of quanta

Based on the description of objects of vacuum by RGWs and by relativity, the emergence of quanta should be derived, see section 5. This derivation can be applied to all objects that propagate at $v = c$, and that exhibit wave properties. Hereby, the universality of the Planck constant can be derived. The value of the Planck constant must be measured, while its universality is derived.

8.8. Deterministic dynamics

Based on the DEQ of RGWs, the Schrödinger equation should be derived, see section 5. Thereby, the additional derivative with respect to time can be motivated by the probabilistic dynamics in Eq. {67}.

8.9. Observables and operators

Based on the Hilbert space, the correspondence between observables and hermitian operators should be introduced.

8.10. Probabilistic dynamics

Based on the DEQ {13} of the formation of vacuum, the probabilistic dynamics in the measurement should be derived in the basic form of Eq. {67}, see section 6.4. Depending on the students and on the available time, the full probabilistic dynamics, Eq. {64}, and the density operator, sections 6.5, can be treated.

8.11. Uncertainty complementarity and entanglement

The subject area of uncertainty, complementarity and entanglement is particularly interesting. As we derived all postulates of quantum physics, and as uncertainty, complementarity and entanglement can be derived from these postulates (e. g. Ballentine 1998, Ma et al. 2016), these three topics can be treated, interpreted and explained within the framework derived of QP (Carmesin et al. 2020), as derived here. Note that quantum field theory exhibits divergencies in an entanglement entropy (Witten 2018), this property of quantum field theory might reflect the fact that quantum field theories are based on a short wavelength-approximation of the present and derived description of quantum physics (Carmesin 2022b).

8.12. Quantum cryptography and computer

The subject area of quantum cryptography and quantum computer is especially relevant for technology and for the personal application of digital tools. As we derived all postulates of quantum physics, and as these two topics can be derived from these postulates (e. g. Ballentine 1998), quantum cryptography (Carmesin et al. 2020) and quantum computers (Burisch et al. 2023) can be treated directly on the basis derived here.

9. Experience with teaching

The scientific question about the correspondence of relativity and quantum physics can be treated in an analytic and productive manner. I presented that concept to learners ranging from class 8 to 13 in a research club (see Carmesin, 2021f) and to students of a general studies course at the university Bremen. In both groups, the students were able to describe the steps of the respective derivations and to discuss the consequences.

In the derivation of the dynamics of vacuum, the students discussed in more detail the expansion of space according to the Friedmann Lemaitre equation, including the accelerated expansion. Perhaps, the students wanted to be very sure about the uniform transformation of space, the corresponding increase of volume and the respective observations. In particular, the students discussed in detail the function $H_0(z)$. Presumably, they wanted to be very sure about the tests of the vacuum dynamics.

Furthermore, the students discussed in detail the difference between the flowing additional vacuum at a location and the new formed vacuum at the same location. This behaviour could be expected, as that difference might easily be overlooked. Also the physical reality of vacuum was discussed extensively. This might be expected as well as that vacuum is usually not discussed or analysed in the transformations of general relativity. Also the frames have been discussed, as they are always discussed in relativity and in mechanics (e. g. for kinetic energy).

Moreover, the students discussed the far distance limit. This could be expected, as it is essential for the linearity versus nonlinearity of the DEQ. Progressively, the students discussed the deterministic dynamics of wave functions, the stochastic dynamics based on the energy density and about the role of the Hilbert space. These topics could be expected, as they are basic to the postulates of quantum physics.

Altogether, the students showed that they can describe the derivations and that they can discuss especially interesting topics within the relation between relativity and quantum physics.

10. Summary

Students are highly interested in astronomy, astrophysics and in the characteristics of nature. An especially interesting, essential and fundamental concept is presented by quantum physics. However, it was not yet really understood (see e. g. Feynman 1965, p. 129). Moreover, the relation of quantum physics and relativity present an interesting scientific question. Thus, that topic includes a substantial didactical potential and an exciting didactical perspective, see Figs. (1, 2).

In order to use that didactical potential, an analysis and a productive concept of the subject are necessary. For it, we presented sections 2-6. In these, we derived quantum physics from the dynamics of vacuum. For it, we derived the dynamics of vacuum from gravity and relativity, and we considered a variety of tests of these dynamics.

Thus, quantum physics is now understood on the basis of a dynamics of vacuum, which includes the dynamics of space and time. So, we achieved a very clear and fundamental derivation and explanation of quantum physics.

That insight should be made available to students of quantum physics. For this purpose, I developed a course in quantum physics that is based on the dynamics of vacuum. The course can easily be supplemented by any desired generalizing or special topics in quantum physics.

I tested parts of the course in a research club (see e. g. Carmesin 2018c, 2019b, 2020b, 2021e, Sawitzki and Carmesin 2021, Schöneberg and Carmesin 2021) in public astronomy evenings in the assembly hall and in online courses at Bremen university. Thereby, the didactical potential of the topic was used, so that students described and discussed the steps of the derivation of the dynamics of vacuum, including the application of these dynamics to quantum physics and cosmology, see Fig. 3.

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Acknowledgement

I am very grateful to I. Carmesin for many interesting discussions and for proofreading the manuscript. I am also grateful to Matthias Carmesin and Daphne Carmesin for additionally proofreading the manuscript.