Explanation of the Rapid Enlargement of Distances in the Early Universe

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Abstract

The expansion of space since the Big Bang, and the evolution of the early universe in particular, are challenging topics for students in research clubs or similar learning groups. Here we study the conservation of energy during that expansion, the origin of the rapid enlargement in the so-called era of 'cosmic inflation' and the origin of the energy and matter in the universe. The results are worked out in full detail here, so that they can be directly used in a learning group.

1. Introduction

The universe has always been interesting to humans (Hoskin 1999). Accordingly students like to take part in astronomy clubs or they choose astrophysical projects in a research club. These interests provide chances for scientific education. Here I outline a corresponding project, and I report about experiences with teaching it.

1.1. Early universe: a lab for quantum gravity

In the early universe, the density was very high, and so gravity was the dominating interaction. Moreover, distances were very small, and so quantum physics is essential. As there are many corresponding observations, the early universe is an ideal lab for the combination (Bronstein 1936) of gravity and quantum physics. Here we treat the following problems.

1.2. Problem: Conservation of energy

The universe expands since the Big Bang. This is described mathematically by a scaling factor k (Slipher 1915, Friedmann 1922, Wirtz 1922, Hubble 1929). Thereby the wavelengths of the photons increase by the same scaling factor. As a consequence, the kinetic energy h/T of the photons decreases. So the principle of the conservation of energy appears to be violated. This principle is essential for physics, as that principle is important and since most energy was represented by photons in the so called radiation era (Planck 2018). Moreover that problem is essential for science education, as the concept of energy is a basic concept (Niedersächsisches Kultusministerium 2015). Here the conservation of energy is derived, and a corresponding observer is specified.

1.3. Problem: Rapid enlargement

In the early universe, there occurred a very rapid increase of distances by a factor of approximately $Z \approx 10^{28}$ in the so-called era of 'cosmic inflation', as discovered by Alan Guth (Guth 1981). However, in his publication Guth also pointed out, that this expansion cannot be explained by usual physical concepts. Later a hypothetic 'inflaton field' has been proposed that could in principle cause that rapid increase (see for instance Nanopoulos1983). However, such a hypothetic new entity cannot serve as an explanation, as an explanation is a consequence of a theory that is well founded (Ruben 1990). Here an explanation is presented in the framework of quantum gravity, a combination of two well founded theories: gravity and quantum physics (Carmesin 2017, Carmesin 2018a-d, Carmesin 2019a-b, Carmesin 2020a-b).

1.4. Problem: Origin of the energy

The universe expands since the Big Bang. Here it is derived that thereby the energy is conserved. So the question arises: What is the origin of that energy. This problem is essential for physics, as the principle of conservation of energy is important. Moreover that problem is relevant for science education, as energy is a basic concept (Niedersächsisches Kultusministerium 2017).

2. Students

The present project has been tested in a research club with students in classes 9 to 12. The students also attend an astronomy club and apply computers. The students present three parallel reports here: They show that the usual dynamics of the expansion of space is insufficient, in order to describe the full dynamics from the light horizon until the Planck length. And they show how a folding of the space to higher dimensions can complete the dynamics. Moreover they simulate the dynamics at a very high density by the Newton Schrödinger equation.

3. Binary fluid in the early universe

In the early universe, the usual massive elementary particles did not yet exist. So the energy was present in the form of photons and in the form of black holes. In principle, these black holes could evaporate by Hawking radiation (Bekenstein 1973, Hawking 1975). However, it has been shown for three dimensional space (Carmesin 2020a) and for $D \geq 3$ dimensional space (Carmesin 2020b) that such evaporation is impossible, whenever the density $\boldsymbol{\rho}$ is larger than

1/9047. Here and in the following, physical quantities in Planck units (see Planck 1899 or for instance Carmesin 2020a) are expressed in bold face letters.

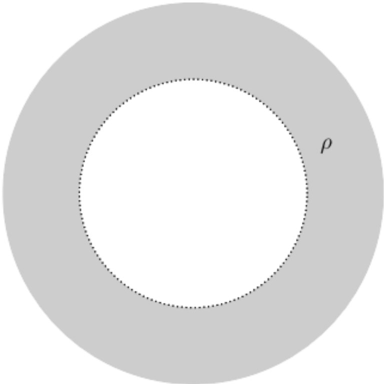

Fig.1: Hollow sphere.

4. Hollow sphere in the fluid

In our analysis we consider the fluid in a sphere. In order to investigate the gravitational influence of the surroundings, we consider a hollow sphere embedded in a homogeneous surrounding of the fluid first (Fig. 1). In such a sphere, the gravitational field is zero, as no matter or energy is included in the sphere. The students can verify this fact by an integration of forces, performed analytically or numerically.

5. Sphere filled with fluid

Next we fill the hollow sphere with fluid, in order to investigate the dynamics of the fluid. For it we consider particles with numbers j and positions $\vec{r}_j$. The dynamics is characterized by a superposition of the chaotic motions and positions $\vec{r}_j$ of the particles, by the averaged motions and positions $[\vec{r}_j]$ and by the expansion of the space.

The chaotic motion can be averaged out by taking the average of positions $\vec{r}_j$. This concept is completely analogous to the concept of the velocity of wind: It is the averaged velocity of the molecules in the air.

In an expanding universe, the averaged positions $[\vec{r}_j](t)$ move outwards as a function of time. In particular, when the expanding space is scaled by a scaling factor k during an interval Δt of time, then the averaged positions are scaled by the same scaling factor:

$$[\vec{r}_j](t+\Delta t) = [\vec{r}_j](t) \cdot k \qquad \{1\}$$

The chaotic motion of the particles generates a corresponding gas pressure. Thereby the gas can be classical like an ideal gas (Clapeyron 1834) or like a van der Waals gas (van der Waals 1873), or the gas can be a quantum gas such as the electron gas in a metal (Fermi 1926). It is possible to derive the dynamics (differential equation, DEQ) of the expansion in terms of the Friedmann Lemaître equation, FLE, without determining the pressure first (Friedmann 1922, Lemaître 1927). We use this approach too.

If the combination of the energy density of the universe with the pressure and the initial momentum of the fluid would generate a positive inward pressure, then the averaged positions would move inwards. This is not the case, and so there is no such effect on the fluid in the sphere.

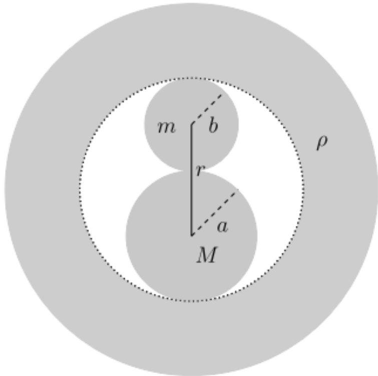

Fig.2: Sphere with inner sphere and particle.

6. Fluid in the sphere

The sphere is filled with particles of the fluid at the actual density **ρ**. We analyze the averaged radii $[\vec{r}_j]$ of these particles. Thereby the particles in the surroundings do not exert a gravitational force nor a hydrostatic force upon the inner particles. So we analyze the inner particles alone.

For it we analyze the distance [r] of a particle from the center of an inner sphere as a function of the time in the framework of general relativity GRT (Fig. 2). We write r instead of [r], for short. An observer at a fixed radius r and with an own time τ (proper time) describes r(τ) and v = dr/dτ as follows (Landau 1981 or Carmesin 2018e, Carmesin 2018f, Carmesin 2020a):

$$E_\infty = E_\infty \cdot (1 - R_S/r)^{1/2} / (1 - v^2/c^2)^{1/2} \qquad \{2\}$$

Thereby R_S is the Schwarzschild radius of the subsphere, and E_∞ is the energy of the mass m at the limit r to infinity. This relation represents the conservation of energy: When the particle moves upwards, then its radius r increases, so the position factor $(1 - R_S/r)^{1/2}$ increases, and the Lorentz factor decreases correspondingly, so that the product is constant. The decreasing Lorentz factor describes the decreasing kinetic energy, while the increasing position factor describes the increase of potential energy.

7. Equivalence to Friedmann Lemaître equation

In this part we show that the dynamics of the fluid (see Eq. {2}) is equivalent to the Friedmann Lemaître equation, FLE, in the q-classical limit. This limit is defined by the limit h to zero, and thereby the single particle m becomes point-like. So the dynamics (DEQ) of the fluid is equivalent to the dynamics (DEQ) of the expansion of space. Thus we develop the macroscopic FLE from the microscopic particles of the fluid. For it we solve Eq. {2} for v^2, and we apply $R_S = 2GM/c^2$:

$$v^2 = 2GM/r \qquad \{3\}$$

We divide by r^2, we introduce the density via $M = \rho \cdot r_1^3 \cdot 4\pi/3$, and we apply the q-classical limit, in which the particle m is point-like, and so we use $r_1 = r$. So we get:

$$v^2/r^2 = 8\pi G/3 \cdot \rho \qquad \{4\}$$

This equation is equivalent to the FLE with zero curvature parameter, as $v = dr/d\tau$.

Thereby the subsphere has been chosen without any restriction. Thus it is representative for the whole fluid. So we conclude:

The microscopic dynamics (DEQ) of the averaged radius [r] of particles in the fluid is equivalent to the macroscopic dynamics (DEQ) of the FLE, in the q-classical limit. This shows that the microscopic rate of increase of the averaged distance r = [r] corresponds to the macroscopic expansion of space.

8. Why is the curvature parameter zero?

In Eq. {3}, v represents the escape velocity of averaged particle positions [r] of the fluid from another averaged particle. So the averaged particle position [r] is at its escape velocity all the time. Why is the velocity not higher than the escape velocity? At the Schwarzschild radius, the velocity is c, so it cannot be higher. In ordinary life, of course, the velocity can be larger than the escape velocity, since objects can be propelled artificially. This is not possible however, if the object starts at R_S.

Why is the velocity not smaller than the escape velocity? The objects described by E_∞ include averaged portions of light that have an average velocity v relative to M. If light does escape from R_S, it can rise from M until the limit r to infinity, as their whole energy is kinetic energy, since their rest mass is zero. So the averaged light always reaches the escape velocity, if it can leave R_S at all.

Of course, also a mass starting at the Schwarzschild radius requires the velocity of light, in order to leave R_S at all, as the position factor is zero at R_S.

As a consequence, the curvature parameter is zero and the space is flat. The present derivation of the curvature parameter zero also solves the flatness problem (Guth 1981).

9. Conservation of energy

In the frame of the observer at fixed r, the microscopic dynamics (DEQ) of the fluid in Eq. {2} represents the conservation of energy. So the macroscopic dynamics (DEQ, FLE) of the expansion of space conserves the energy. For it the following frame should be used: When the time evolution of a sphere of space is observed, then the frame should be located at a fixed distance r to the Schwarzschild radius of that sphere.

10. Quantization

Next we investigate two particles locally at a density ρ. Thereby we analyze the ground state. Physically this means that energy may be exchanged with the fluid, thus the fluid may be considered as a heat bath. So this investigation is quite different from the above analysis of q-classical and global energies.

We quantize the microscopic dynamics of Eq. {3}, corresponding to the macroscopic dynamics. For it we multiply that Eq. with the mass or dynamic mass $m = E_\infty/c^2$, we divide by 2, and we subtract the term at the right hand side of the Eq.:

$$\tfrac{1}{2}\, m \cdot v^2 - G \cdot M \cdot m/r = 0 \qquad \{5\}$$

We realize that this term represents the energy E of the particle m corresponding to the dynamics (DEQ) of the expansion of space. Moreover we express the velocity v by the momentum $p = m \cdot v$:

$$\tfrac{1}{2} \cdot p^2/m - G \cdot M \cdot m/r = E = 0 \qquad \{6\}$$

We quantize this DEQ by replacing the observables by the corresponding operators (see for instance Ballentine 1998). Moreover we apply the expectation value:

$$\tfrac{1}{2} \cdot \langle p^2 \rangle /m - G \cdot M \cdot m \cdot \langle r^{-1} \rangle = \langle E \rangle \qquad \{7\}$$

The energy in Eq. {6} looks like a classical energy, though it has been derived from GRT without any approximation. This can be understood in the framework of an equivalence principle (Carmesin 2019a-b).

11. Ground state energy

In this investigation, we analyze the ground states, as these provide realistic information, since the gravity was very strong in the early universe and dominated other forms of energy. We derive the ground state as follows: In Eq. {7} we apply the identity $r^{-1} = (r^2)^{-0.5}$. Furthermore we use the approximation $\langle (r^2)^{-0.5} \rangle \approx (\langle r^2 \rangle)^{-0.5}$.

Moreover we use the mathematical identity:

$$\langle x^2 \rangle = \langle x \rangle^2 + (\Delta x)^2, \qquad \{8\}$$

So we get:

$$\langle E \rangle = \langle p \rangle^2 / (2m) + (\Delta p)^2 /(2m) - G \cdot M \cdot m \cdot (\langle r \rangle^2 + (\Delta r)^2)^{-0.5}$$

Here we expand in linear order in $(\Delta r)^2/\langle r \rangle^2$. So we obtain:

$$\langle E \rangle = \langle p \rangle^2 /(2m) + E_{cl,G} + E_Q \qquad \{9\}$$

Hereby $E_{cl,G}$ is the non-quantum gravity term

$$E_{cl,G} = - G \cdot M \cdot m/\langle r \rangle \qquad \{10\}$$

and E_Q is the additional quantum term, for the case $\langle p \rangle = 0$:

$$E_Q = (\Delta p)^2/(2m) + \tfrac{1}{2}\, G \cdot M \cdot m \cdot (\Delta r)^2/\langle r \rangle^3 \qquad \{11\}$$

For Gaussian wave functions we obtain from the uncertainty relation the minimal uncertainty:

$$\Delta x \cdot \Delta p = \tfrac{1}{2} \cdot \underline{h} = h/(4\pi) \qquad \{12\}$$

With it we get:

$$E_Q = \underline{h}^2/[8m \cdot (\Delta r)^2] + \tfrac{1}{2}\, G \cdot M \cdot m \cdot (\Delta r)^2/\langle r \rangle^3 \qquad \{13\}$$

12. Higher dimension

At high density, gravity is very strong and tends to make objects very compact. Examples are white dwarfs, neutron stars and black holes. Another example for a very compact object is a parachute: in the unfolded state it is practically two dimensional and large, while in the folded state it is three dimensional and small. This example shows that compact objects can be generated by an increase of the dimension. Dimensions larger than three have already been observed: In two different experiments the four-dimensional quantum Hall effect has been put into effect (Lohse et al. 2018; Zilberberg 2018 et al.). Accordingly, we generalize our model to dimensions $D \geq 3$. For it, the potential energy term

$$E_{pot} = -G \cdot M \cdot m / < r > \qquad \{14\}$$

is replaced by the following term:

$$E_{pot} = -G \cdot L_P^{D-3} \cdot M \cdot m / < r^{D-2} > \qquad \{15\}$$

The exponent $D - 2$ is a consequence of Gaussian gravity (Gauss 1813; Bures 2011), and the factor L_P^{D-3} can be derived by using the concept of the Schwarzschild radius (see Carmesin 2017, Carmesin 2019, Carmesin 2020a,b). With it we generalize equation {10}:

$$E_{D,cl,G} = -G \cdot L_P^{D-3} \cdot M \cdot m / < r >^{D-2} \qquad \{16\}$$

Here and in the following, we mark the dependence of the energy on D by a subscript. In D dimensional isotropic space, the uncertainty relation {12} is:

$$\Delta x \cdot \Delta p = ½ \cdot D^{1/2} \cdot \underline{h} \qquad \{17\}$$

So the quantum term is generalized as follows:

$$E_{D,Q} = D \cdot \underline{h}^2 / [8m \cdot (\Delta r)^2]$$
$$+ ½ \cdot (D-2) \cdot G \cdot L_P^{D-3} \cdot M \cdot m \cdot (\Delta r)^2 / < r >^D \qquad \{18\}$$

13. Planck units

In order to simplify the above equations, we use Planck units, we use the normalized energy $\underline{E} = E/(m \cdot c^2)$, and we apply equation {4}. So we get:

$$\underline{E}_{D,cl,G} = -\mathbf{M} / < \mathbf{r} >^{D-2} \qquad \{19\}$$

Similarly we obtain:

$$\underline{E}_{D,Q} = D / [8\mathbf{m}^2 \cdot (\Delta \mathbf{r})^2]$$
$$+ (D-2) \cdot (\Delta \mathbf{r})^2 \cdot \mathbf{M} / (2 \cdot < \mathbf{r} >^D) \qquad \{20\}$$

14. Quantum fluctuations

Here we investigate the case in which small particles have not yet formed. The students determine the quantum fluctuations $q = (\Delta \mathbf{r})^2$ by application of the variational method. So they derive the value of q that minimizes the above term for the quantum energy $\underline{E}_{D,Q}$. So they get:

$$(\Delta \mathbf{r})^4 = D \cdot < \mathbf{r} >^D / [4(D-2) \cdot \mathbf{m}^2 \cdot \mathbf{M}] \qquad \{21\}$$

We insert this term into the above term for E_Q. So we obtain the ground state quantum energy:

$$\underline{E}_{D,Q} = [D \cdot (D-2) \cdot \mathbf{M}]^{1/2} / (2 \cdot \mathbf{m} \cdot < \mathbf{r} >^{D/2}) \qquad \{22\}$$

The normalized energy is:

$$\underline{E}_D = \underline{E}_{D,Q} + \underline{E}_{D,cl,G} = E_D \qquad \{23\}$$

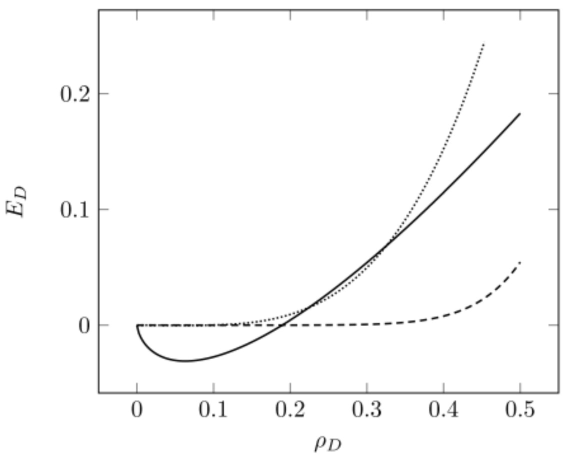

Fig.3: Normalized energies E_D as a function of the density $\boldsymbol{\rho}_D$ for the case of a photon with dynamical mass M and a black hole with mass m. At the Planck density: $\boldsymbol{\rho}_D = ½$. Solid line (D = 3), dotted (D = 7) and dashed (D = 15).

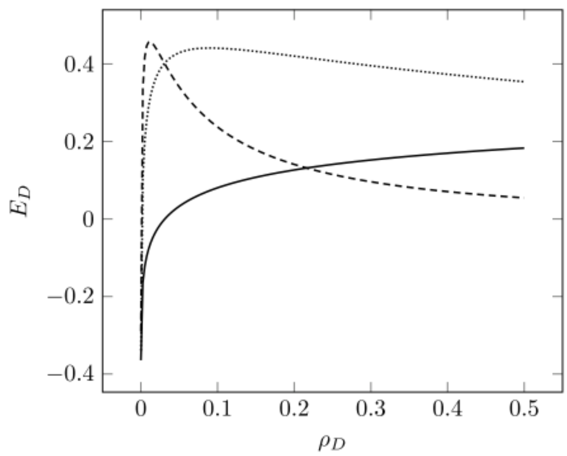

Fig.4: E_D ($\boldsymbol{\rho}_D$) for the case of a photon with dynamical mass m and a black hole with mass M. Solid line (D = 3), dotted (D = 7) and dashed (D = 15).

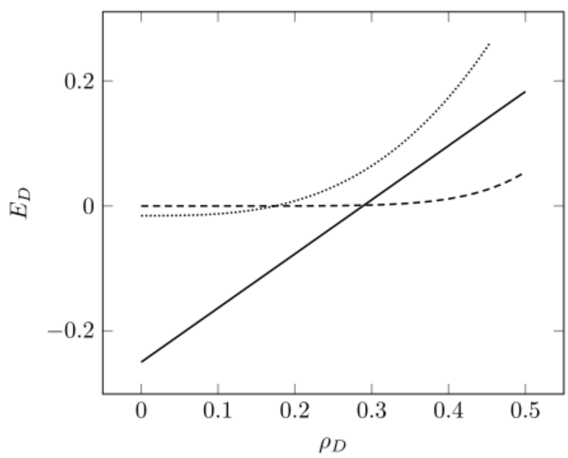

Fig.5: E_D ($\boldsymbol{\rho}_D$) for the case of two black holes. Solid line (D = 3), dotted (D = 7) and dashed (D = 15).

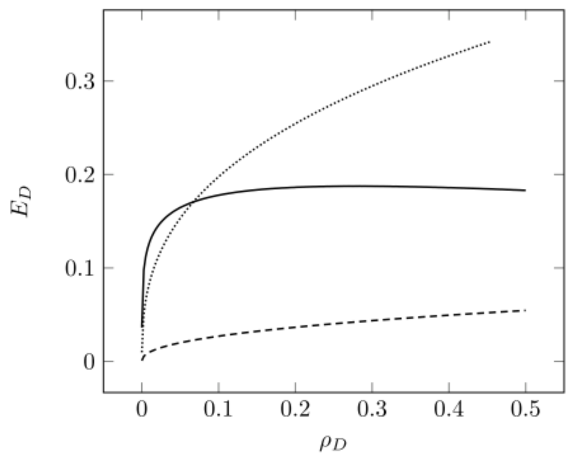

Fig.6: E_D (**ρ_D**) for the case of two photons. Solid line (D = 3), dotted (D = 7) and dashed (D = 15).

15. Dimension at minimal E_D

Here we investigate the case, in which m and M represent masses or dynamical masses of particles of the fluid. The term E_D represents the energy of a particle m including the interaction with a neighboring particle M. In principle, there are four possibilities: M and m both represent two photons or two black holes, M is a photon and m a black hole and vice versa. Here we show that in all four cases, the energy becomes minimal, if the system increases its dimension D in a dimensional transition.

The numeric analysis shows: at high density, the normalized energy E_D for D = 3 is not optimal (see Figs. 3-6). In contrast, higher dimensional space is stable at high density. As a consequence, the space folds to high dimensions at high density. Conversely, in the case of the Big Bang, the universe starts at high density. When the density decreases, then critical densities are reached at which the dimension is reduced. Thereby the distances increase dramatically by a factor of approximately $Z \approx 10^{28}$. This process has been worked out in detail in (Carmesin 2017, Carmesin 2019a-b, Carmesin 2020a-b).

16. Origin of the energy

The kinetic energy of radiation h/T tends to ≈ 0, as the distance [r] tends to infinity, as the wavelengths increase correspondingly. Accordingly, the respective energies in Eq. {2} tend to ≈ 0. In particular, Eq. {2} is transferred to photons as follows:

$E_\infty = (1 - R_S/r)^{1/2} \cdot h/T(r) = h/T_\infty = \text{constant}$ {24}

With the periodic time:

$T(r) = T_\infty \cdot (1 - R_S/r)^{1/2}$ {25}

The usual elementary particles are directly or indirectly generated by photons in reactions that conserve the energy, and the dynamics of the particles is characterized by Eq. {2}. So the corresponding energy $E_{kin}(r)/E_{kin}(R_S)$ tends to zero at [r] to infinity.

However, the dark energy is a source of accelerated expansion. So that energy appears to be not conserved. But the dark energy represents the space, can be modeled by a zero point oscillation, ZPO, and so it cannot be converted into any other physical entity. So the corresponding zero point energy, ZPE, does not need energy conservation. In this sense the energy is conserved, as the ZPE does not need any source of energy. In this manner, the origin of the energy is explained:

Radiation originates from its ZPE at the original state at L_P, hence it is at the velocity of flight, and thus it provides zero curvature. At the dimensional transitions, that ZPE becomes available energy, and it partially becomes gravitational energy according to the gravitational position factor $(1 - R_S/r)^{1/2}$. Matter originates from radiation. The ZPE of the dark energy does not need any source, as a ZPO cannot be converted without destroying the corresponding system. The time evolution of these zero point energies precisely explains the various densities observed today (Carmesin 2017, 2018a-d, 2019a,b).

17. Discussion

The questions of the conservation of the energy, the origin of the energy and of the so-called ‘cosmic inflation’ have been addressed in a research club. This report presents an overview, while students report about numerical studies related to these questions. The achieved results are presented in detail and can be used in classes or courses.

We derive a mathematical and partially physical correspondence of the microscopic dynamics of particles to the macroscopic dynamics of space. With it we transfer the conservation of energy from the microscopic dynamics to the macroscopic dynamics, in spite of the loss of kinetic energy h/T of photons as a consequence of the redshift.

With that dynamics we quantize the microscopic fluid and apply that quantization to the very early universe. With it, we derive a gravitational instability: At high density, the space is folded to higher dimension. This explains the rapid enlargement in the early universe qualitatively and quantitatively.

Additionally, we analyze the origin of the energy: The present energies of radiation and matter originate from the ZPE of radiation at L_P, providing zero curvature as a consequence. The kinetic energy h/T is partially transformed to gravitational energy. In other words: The present state in which we live is characterized by a negative gravitational potential energy, and the difference to the original ZPE of radiation is present in the form of radiation, planets, stars and so forth via the equivalence of mass and energy, $E = m \cdot c^2$. The dark energy has been explained earlier by a ZPO that cannot be transformed, and it does not need a source of energy therefore (Carmesin 2018a-d, Carmesin 2019a,b).

The explanation of a physical phenomenon based on competences of the students provides a high learning efficiency (Hattie 2009, Kircher 2001), so it is very useful in science education.

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